S.6 PURE MATHEMATICS ASSIGNMENT (1)

Paper 1

INSTRUCTIONS TO CANDIDATES

- *Answer* **all** *questions in both sections* ***A*** *and* ***B.***

SECTION A

1. Given that $p^2 = qr$, show that;

$2\log_q p \log_r p = \log_q p + \log_r p$ $(05\ marks)$

2. Evaluate;

$$\int_1^4 \frac{\left(1+\sqrt{x}\right)^5}{\sqrt{x}}\,dx$$ $(05\ marks)$

3. Find the equations of the tangents to the parabola $y^2 = 6x$ which pass through the point $(10\,,-8)$.

$(05\ marks)$

4. Solve the equation

$\sqrt{x+5} + \sqrt{x+21} = \sqrt{6x+40}$ $(05\ marks)$

5. If $y = 3^x$, find $\frac{d^2y}{d\,x^2}$ when $x = -1$. $(05\ marks)$

6. Solve the simultaneous equations

$$\cos x + \cos y = 1$$

$$\sec x + \sec y = 4$$

for $0^\circ < x, y < 180^\circ$ $(05\ marks)$

7. Show that the lines $\boldsymbol{L_1}$, vector equation $\boldsymbol{r} = \begin{pmatrix}2\\5\end{pmatrix} + \lambda\begin{pmatrix}2\\-3\end{pmatrix}$ and $\boldsymbol{L_2}$, vector equation $\boldsymbol{r} = \begin{pmatrix}3\\-3\end{pmatrix} + \mu\begin{pmatrix}3\\2\end{pmatrix}$ are perpendicular and find the position vector of their point of intersection . $(05\ marks)$

8. Find the percentage increase in the volume of a cube when all the edges of the cube are increased by 2 % . $(05\ marks)$

SECTION B (60 MARKS)

9. (a) Given that Z_1 and Z_2 are complex numbers , solve the simultaneous equations;

$$4Z_1 + 3Z_2 = 23$$

$$Z_1 + iZ_2 = 6 + 8i$$

$(06\ marks)$

(b) If $Z = x + yi$, find the locus given by;

$$\left|\frac{Z - 1}{Z + 1 - i}\right| = \frac{2}{5}$$

$(06\ marks)$

10. Given that $x = \frac{1-t}{1+t}$ and $y = (1-t)(1+t)^2$, find $\frac{d^2y}{d\,x^2}$.

$(12\ marks)$

11.(a) Find the coordinates of the foot of the perpendicular from the point $(2, -6)$ to the line $3y - x + 2 = 0$. $(04\ marks)$

(b) A circle touches both the $x -$ axis and the line $4x - 3y + 4 = 0$ .Its centre is in the first quadrant and lies on the line $x - y - 1 = 0$. Prove that its equation is $x^2 + y^2 - 6x - 4y + 9 = 0$. $(08\ marks)$

12. Express $y = \frac{64x^2 - 148\,x + 78}{(4\,x - 5)^3}$ into partial fractions hence find $\int_4^6 y\,dx$.

$(12\ marks)$

13. Given that $y = \frac{\cos x - 2\cos 2x + \cos 3x}{\cos x + 2\cos 2x + \cos 3x}$, Prove that $y = -tan^2\frac{x}{2}$ hence find the value of $tan^2 15^\circ$ in the form $p + q\sqrt{r}$ where p, q and r are integers and solve;

$2y + sec^2\frac{x}{2} = 0$ for $0^\circ \leq x \leq 360^\circ$ $(12\ marks)$

14. (a) Solve the differential equation;

$\frac{dy}{dx} = \sqrt{\frac{y}{x+1}}$ given that $y = 9$ when $x = 3$ $(05\ marks)$

(b) At time t minutes , the rate of change of temperature of a cooling liquid is proportional to the temperature , $T^\circ\ C$ of that liquid at that time . Initially $T = 80$

(i) Show that $T = 80e^{-kt}$

(ii) If $T = 20$ when $t = 6$, find the time at which the temperature will reach $10^\circ C$. $(07\ marks)$

15 . (a) One root of the equation $x^2 - 6x + k = 0$ is three times the other. Find the roots and the value of k . $(05\ marks)$

(b) If x is so small that x^5 and higher powers of x can be neglected such that

$(1 + x)^6(1 - 2x^3)^{10} \approx 1 + ax + bx^2 + cx^4$.

Find the values of a , b and c . $(07\ marks)$

16. (a) Find the Cartesian equation of the plane containing the point $(1,3,1)$ and parallel to the vectors $\boldsymbol{i} - \boldsymbol{j} + 3\boldsymbol{k}$ and $2\boldsymbol{i} + \boldsymbol{j} - 3\boldsymbol{k}$. $(07\ marks)$

(b) Find the angle between the plane in (a) above and the line $\frac{x-6}{5} = \frac{y-1}{-1} = \frac{z+1}{4}$ $(05\ marks)$

END