S5 PHYSICS PAPER TWO OPTICS (continuation)

1. A convex mirror whose radius of curvature is 30cm forms an image of a real object which has been placed 20cm from the mirror. Calculate the position of the image and the magnification produced.
   Solution:
   Using $\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \rightarrow \frac{1}{20}+\frac{1}{v}=\frac{1}{-15} \rightarrow v=\frac{-60}{7}=-8.5714cm$
   $m=\frac{8.5714}{20}=0.43$
2. A concave mirror with radius of curvature 40cm forms an image of a real object which has been places 25cm from the mirror.
   (a) What is the focal length of the mirror
   (b) Calculate the distance of the image from the mirror and state whether it is real or virtual.
   Solution:
   (a) $f=\frac{40}{2}=20cm$
   (b) $\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \rightarrow \frac{1}{25}+\frac{1}{v}=\frac{1}{20} \rightarrow v=100cm$
   The image is real since v is positive.
3. What is the focal length of a mirror that forms a 4 times magnified image of an object which has been placed 20cm from the mirror. State whether the mirror is concave or convex.
   Solution:
   $m=\frac{v}{u} \rightarrow 4=\frac{v}{20} \rightarrow$ v = 80cm.
   But $\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \rightarrow \frac{1}{20}+\frac{1}{80}=\frac{1}{f} \rightarrow f=40cm$
   It is a concave mirror because it has a positive focal length.
4. A concave mirror forms a real image which is 3 times the linear size of the real object. When the object is displaced through a distance, d, the image formed is 4 times the linear size of the object. If the distance between the two images is 20cm, find;
   (i) The focal length of the mirror
   (ii) Distance, d.
   Solution:
   (i) $m_1=3,\ m_2=4,\ \ v_2-v_1=20cm,\ u_1-u_2=d$
   Using $m=\frac{v}{f}-1 \rightarrow 3=\frac{v_1}{f}-1$ …………….. (i)
   Also $4=\frac{v_2}{f}-1$ …………….. (ii)
   $(ii)=(i)$
   $1=\frac{v_2}{f}-1-(\frac{v_1}{f}-1\ ) \rightarrow 1=\frac{v_2-v_1}{f} \therefore f=20cm$
   (ii) Using $\frac{1}{m}=\frac{u}{f}-1 \rightarrow \frac{1}{3}=\frac{u_1}{20}-1 \rightarrow u_1=26.67cm$
   Also $\frac{1}{m}=\frac{u}{f}-1 \rightarrow \frac{1}{4}=\frac{u_2}{20}-1 \rightarrow u_2=25cm$
   But $u_1-u_2=d \rightarrow d=26.67-25=1.67cm \therefore d=1.67cm$

5. An object placed ucm from a concave mirror forms an image which is 3 times its object. The object is moved by 2cm and the image is 5times the size of the object. Find.
   (i) The focal length of the mirror
   (ii) The object distance.

   Solution:

   (i) $m_1 = 3,\ m_2 = 5,\ u_1 = u,\ u_2 = u - 2$

   Using $\frac{1}{m} = \frac{u}{f} - 1 \ \rightarrow \frac{1}{3} = \frac{u}{f} - 1$ ………………… (i)

   Also $\frac{1}{5} = \frac{u-2}{f} - 1$ …………………….. (ii)

   Substituting (i) into (ii)

   $\frac{1}{5} = (\frac{1}{3} + 1) - \frac{2}{f} - 1 \ \rightarrow \frac{1}{5} - \frac{4}{3} + 1 = -\frac{2}{f} \rightarrow f = 15cm$

   (ii) Using $\frac{1}{3} = \frac{u}{f} - 1 \ \rightarrow \frac{1}{3} = \frac{u}{15} - 1 \ \rightarrow u = 20cm$

6. An object placed in front of a concave mirror forms an image which is 5 times smaller than the object. When the object is moved a distance, dcm, the image formed is 4 times smaller than the object. If the distance between the positions in the two cases is 1cm, find;
   (i) The focal length of the mirror
   (ii) The value of d.

   Solution:

   (i) $m_1 = \frac{1}{5}\ \ m_2 = \frac{1}{4}\ v_2 - v_1 = 1$

   Using $m_1 = \frac{v_1}{f} - 1 \rightarrow \frac{1}{5} = \frac{v_1}{f} - 1$……………. (i)

   Also $m_2 = \frac{v_2}{f} - 1 \rightarrow \frac{1}{4} = \frac{v_2}{f} - 1$……………. (ii)

   $(ii) - (i)$

   $\frac{1}{4} - \frac{1}{5} = \frac{v_2}{f} - 1 - (\frac{v_1}{f} - 1) \ \rightarrow \frac{1}{20} = \frac{v_2 - v_1}{f} \rightarrow \frac{1}{20} = \frac{1}{f} \ \rightarrow f = 20cm$

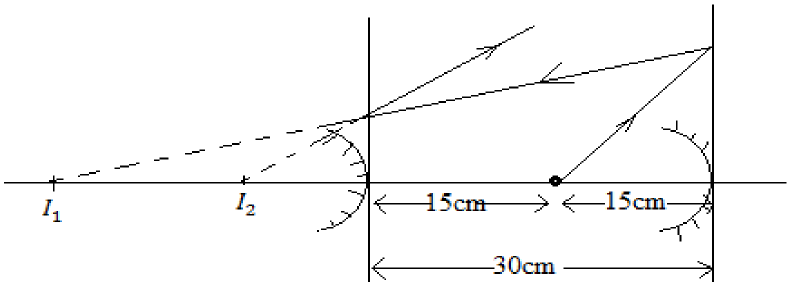
   (ii) $\frac{1}{m_1} = \frac{u_1}{f} - 1 \ \rightarrow \frac{1}{\frac{1}{5}} = \frac{u_1}{20} - 1 \ \rightarrow u_1 = 120cm$

   $\frac{1}{m_2} = \frac{u_2}{f} - 1 \ \rightarrow \frac{1}{\frac{1}{4}} = \frac{u_2}{20} - 1 \ \rightarrow u_2 = 100cm$

   $u_1 - u_2 = d \ \rightarrow d = 120 - 100 = 20cm$

7. A concave mirror of radius of curvature 25cm faces a convex mirror of radius of curvature 20cm and is 30cm from it. If an object is placed midway between the mirrors, find the position and nature of the image formed by the reflection first at the concave mirror and then at the convex mirror.

   Solution:



Consider the action of a concave mirror

From $\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \rightarrow \frac{1}{15}+\frac{1}{v}=\frac{1}{12.5} \rightarrow v=75cm$

Consider the action of a convex mirror

$u=-(75-30)=-45cm$ (Virtual image distance to the convex mirror)

$f=-10cm$

From $\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \rightarrow \frac{1}{-45}+\frac{1}{v}=\frac{1}{-10} \rightarrow v=\frac{-90}{7}=-12.86cm$

∴ The image is virtual and 12.86cm from the convex mirror.

8. A concave mirror M of focal length 20cm is placed 90cm in front of a convex mirror N of focal length 12.5cm. An object is placed on the common axis of M and N at a point 25cm in front of M. Determine the distance from N to the image formed by reflection first in M then in N.

Solution:

Consider the action of a concave mirror

From $\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \rightarrow \frac{1}{25}+\frac{1}{v}=\frac{1}{20} \rightarrow v=100cm$

Consider the action of a convex mirror

$u=-(100-90)=-10cm$ (Virtual image distance to the convex mirror)

$f=-10cm$

From $\frac{1}{u}+\frac{1}{v}=\frac{1}{f} \rightarrow \frac{1}{-10}+\frac{1}{v}=\frac{1}{-12.5} \rightarrow v=50cm$

∴ The image is real and 50cm from the convex mirror.

9. A small convex mirror is placed 60cm from the pole and on the axis of a large convex mirror of radius of curvature 200cm. the position of the convex mirror is such that a real image of a distant is formed in the plane of a hole drilled through the concave mirror at its pole.

(a) Draw a ray diagram to show the formation of the above image

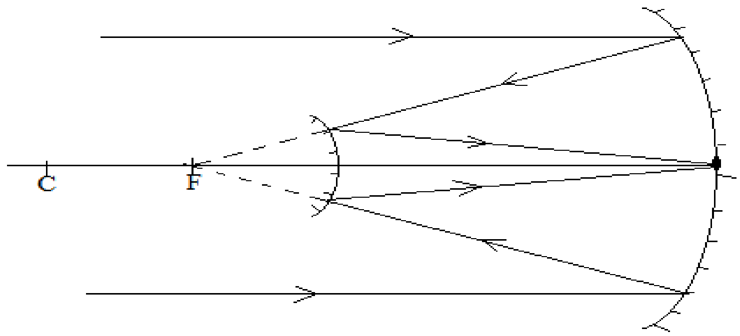
(ii) Suggest a practical application of the arrangement of the mirror in a(i) above.

(b) Calculate the radius of curvature of the mirror.

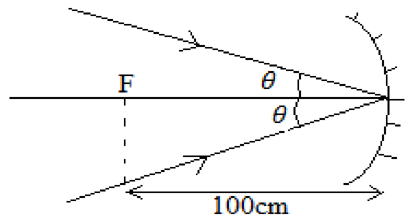
(ii). Height of the real image if the distant object subtends an angle of 0.5^0 at the pole of the convex mirror.

Solution:

Note: the image formed by a concave mirror acts as a virtual object to the convex mirror



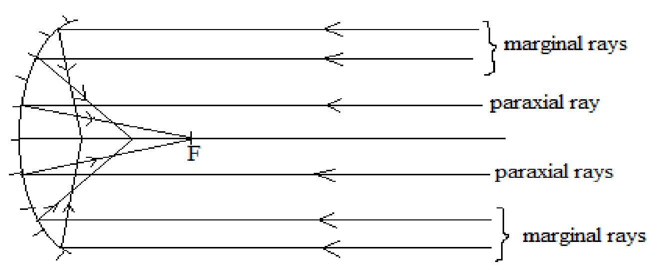
(ii). It is applied in reflecting telescope, a device used for viewing a distant object.
(b). action of a concave mirror, image distance = 100cm
Action of a convex mirror
Image distance $u = -(100 - 60) = -40cm, image\ distance, v = 60cm$
From $\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \rightarrow \frac{1}{-40} + \frac{1}{60} = \frac{1}{f} \rightarrow f = -120cm$
But $r = 2f \quad \rightarrow r = 2(-120) = -240cm$. Thus the radius of the convex mirror = 120cm.
(b)(ii) Height of the image.
Let h_1 be the height of the image formed by a concave mirror.



$\tan\theta = \frac{h_1}{100} \rightarrow h_1 = 100\tan\theta \rightarrow h_1 = 100\tan 5 = 0.8727cm$
Let h_2 be the height of the image formed by a convex mirror.
But $m = \frac{h_2}{h_1} = \frac{v}{u} \rightarrow h_2 = \frac{v}{u}.h_1$ therefore $h_2 = \frac{60}{40} \times 0.8727 = 1.3cm$

REFLECTION OF PARALLEL WIDE BEAM OF LIGHT AT CURVED MIRRORS.

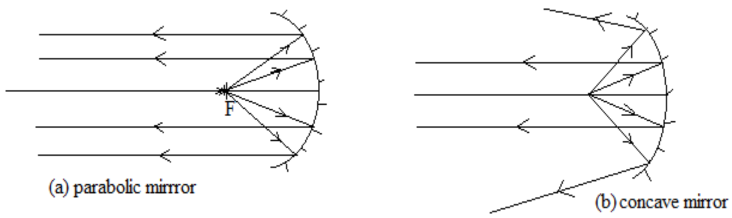
Consider the reflection of a wide parallel beam of light incident on a concave mirror as shown below;



Rays close and parallel to the principal axis converge at F after reflection. When a wide beam of light is incident on a concave mirror, the different reflected rays are converged to different points. However these reflected rays appear to touch a surface known as a caustic surface (a surface on which every reflected ray from the mirror forms a tangent to) and has an apex at the principal focus, F.

Note: the marginal rays furthest from the principal axis are converged nearer to the pole of the mirror than the central rays.

COMPARISION BETWEEN A CONCAVE AND A PARABOLIC MIRROR.



When a lamp is placed at the principal focus of a concave mirror, only rays from the lamp that strike the mirror at points close to the principal axis will be reflected parallel to the principal axis. Those that strike at points far away from the principal axis will be reflected in different directions and not as parallel beam of light as seen in (b) above. In this case the intensity of reflected beam practically diminishes as the distance from the mirror increases.

When a lamp is placed is placed at the principal focus of a parabolic mirror, all rays from the lamp that strike the mirror at points close to or far away from the principal axis will be reflected parallel to the principal axis as seen in (a) above. In this case the intensity of the reflected beam remains practically undiminished as the distance from the mirror increases. This accounts for the use of parabolic mirrors as search light other than a concave mirror.

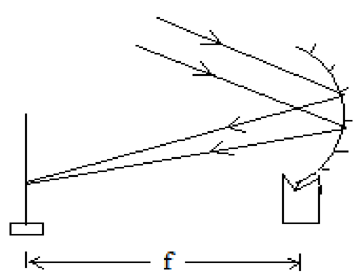
Note: a parabolic mirror has the advantage of reflecting rays from a light source placed at its principal focus as parallel beam of beam with undiminished intensity.

Uses of parabolic mirrors.

(i) They are used as reflectors in search lights, car head lamps etc.

DETERMINATION OF FOCAL LENGTH OF A CONCAVE MIRROR.

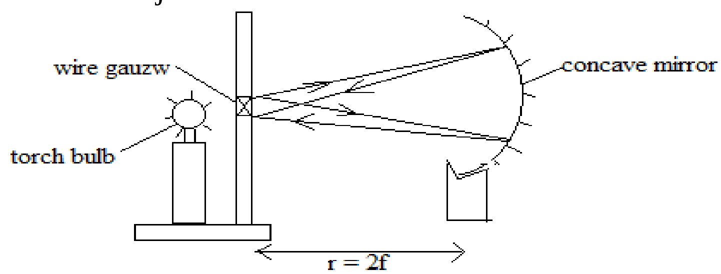
1. Using a distant object.



The setup is made in such a way that the concave mirror faces a distant object.
The position of the screen in front of the mirror is adjusted until a clear image of the distant object is formed on the screen.

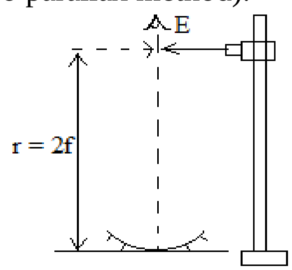
The distance between the screen and the mirror is measured and recorded and it is the focal length of the mirror.

2. Using illuminated object.



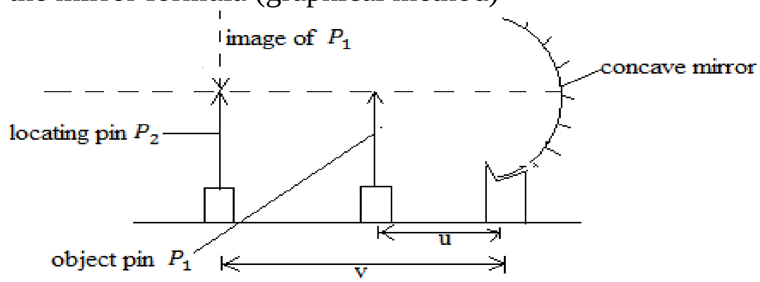
An object in this case consists of a cross wire in a hole on a white screen.
The wire gauge (object) is illuminated from behind by a source of light
A concave mirror mounted in a holder is moved to and fro in front of the screen until a sharp image of the cross wire is formed on the screen adjacent to the cross wire (object).
At this point, both the image and the object are at the same distance from the mirror hence both are at the centre of curvature
The distance between the mirror and the screen is measured and this is the radius of curvature, r. but r = 2f where f is the focal length implying $f = \frac{r}{2}$ hence the focal length, f, can be calculated.

3. Using an object at C (No parallax method).



A concave mirror is placed on a horizontal table with its reflecting surface facing up.
A pin is clamped horizontally on the retort stand such that its pointed end lies along the principal axis of the mirror.
The pin is moved vertically to and fro until a point is located where the pin coincides with its own image.
The distance, r, from the pole to the pin is measured and recorded
The focal length $f = \frac{r}{2}$

4. Using the mirror formula (graphical method)



An object pin P_1 is placed at a distance, u in front of a mounted concave mirror such that its tip lies along the principal axis of the mirror and it forms an inverted image.
A locating pin P_2 is places behind the object pin P_1 and its position is adjusted until it coincides with the image of P_1 by no parallax method.
The distance, v of P_2 from the pole is measured and recorded
The experiment is repeated for several values of u and the corresponding values of v is measured and recorded.
The results are tabulated including the values of uv and $u + v$
A graph of uv against $u + v$ is plotted and its slope, S determined.
The focal length, f of the mirror is got from; f = slope, s
Note:

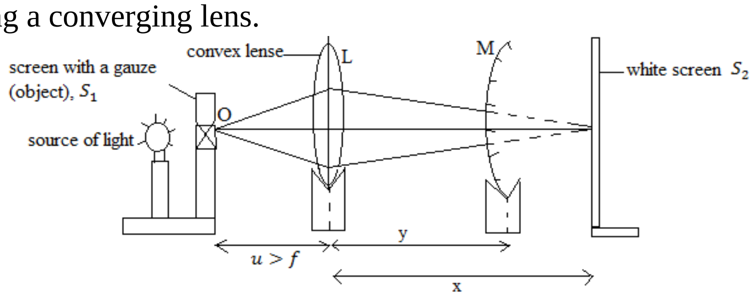
(i) If a graph of $\frac{1}{u}$ against $\frac{1}{v}$ is plotted, the intercept, C on either $\frac{1}{u}$ or $\frac{1}{v}$ is equal to $\frac{1}{f}$ hence f = $\frac{1}{C}$

(ii) If a graph of $u + v$ against uv is plotted, it gives a slope, S = $\frac{1}{f}$ hence f = $\frac{1}{S}$

(iii) If a graph of magnification, m against v is plotted, it gives a slope, S = $\frac{1}{f}$ hence $f = \frac{1}{S}$.

AN EXPERIMENT TO DETERMINE THE FOCAL LENGTH OF A CONVEX MIRROR.

1. Using a converging lens.



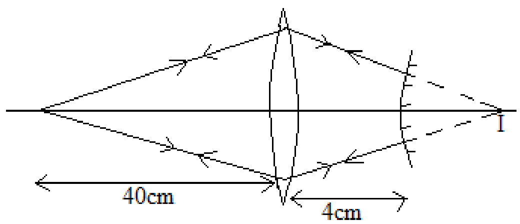
Without a convex mirror, M, the lens L is placed at a distance, u greater than its focal length.
The screen S_2 is moved until a sharp image I is formed on it.
The distance x between L and S_2 is measured and recorded.
The convex mirror is placed between L and S_2 and its position is adjusted until the sharp image of O coincides with O.
The distance, y between L and M is measured and recorded
The focal length of M is then calculated from $f = \frac{x-y}{2}$.

Note: when an incident ray from the object is reflected back along the incident path, a real inverted image is formed besides the object. In this case the rays strike the mirror normally. Therefore, if produced backwards, they will pass through the centre of curvature of the mirror thus the distance MS_2 is the radius of curvature of the mirror, M.

Example:

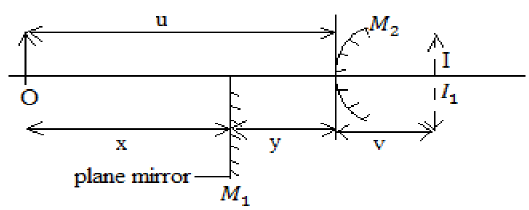
1. An object, O is placed 40cm in front of a convex lens of focal length 15cm forming an image on the screen. A convex mirror situated 4cm from the lens in the region between the lens and the screen forms the final image besides the object, O.
   (i) Draw a ray diagram to show how the final image is formed.
   (ii) Determine the focal length of the convex mirror.
   Solution:

(i) A ray diagram is as below



(ii) Consider the object, lens and the screen only.
From $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ → From $\frac{1}{40}+\frac{1}{v}=\frac{1}{15}$ → $v = 24cm$
Focal length $f = \frac{24-4}{2} = 10cm$
Therefore the focal length of the convex mirror $is\ -10cm$ (a convex mirror has a virtual radius of curvature hence a virtual focal length).

2. Using a plane mirror and no parallax method.



An object pin, O is placed in front of a convex mirror M_2 such that it forms a virtual inverted image at I
The distance of the object, O from the convex mirror is measured and recorded.
A plane mirror M_1 is placed between the object, O and the convex mirror such that it covers half the field of view of the convex mirror
The plane mirror M_1 is adjusted until the image of the object, O through it coincides with I by no parallax.
The distance x and y are measured and recorded.
The focal length f is then calculated from $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$ where $u = (x + y)\ and\ v = -(x - y)$

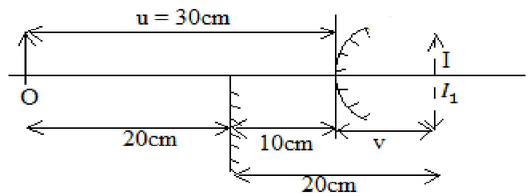
Note:

(i) The two images coincides when they are as far behind the plane mirror as the object is in front.

(ii) Substituting for $u = (x + y)$ and $v = -(x - y)$ in the mirror formula gives $f = \frac{y^2 - x^2}{2y}$

Examples:

1. A plane mirror is placed 10cm in front of a convex mirror so that it covers half of the mirror surface. A pin 25cm in front of a plane mirror gives an image which coincides with that of the pin in the convex mirror. Find the focal length of the convex mirror.
   Solution:
   $u = (25 + 10) = 35cm\ and\ v = -(25 - 10) = -15cm$
   From $\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \rightarrow \frac{1}{35} + \frac{1}{-15} = \frac{1}{f} \rightarrow f = \frac{-105}{4} = -26.25cm$
2. A plane mirror is placed 10cm in front of a convex mirror so that it covers half of the mirror surface. A pin 20cm in front of the plane mirror gives an image which coincides with that of the pin in the convex mirror. Find the focal length of the convex mirror.
   Solution:



$u = (20 + 10) = 30cm\ and\ v = -(20 - 10) = -10cm$
From $\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \rightarrow \frac{1}{30} + \frac{1}{-10} = \frac{1}{f} \rightarrow f = -15cm$

REFRACTION OF LIGHT.

Refraction is the change in the direction of light rays when it travels from one medium to another of different optical densities.

OR: is the bending of light rays as it travels from one medium to another of different optical densities.

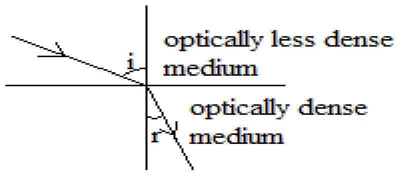
Explanation of refraction:

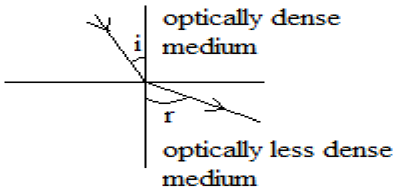
The bending of light rays is as a result of the change in the speed of light as it travels from one medium to another.

The change in the speed of light leads to the change in the direction unless the ray is incident normally.

The speed of light in air is higher than the speed of light in glass and water. Glass and water are therefore said to be denser than air.

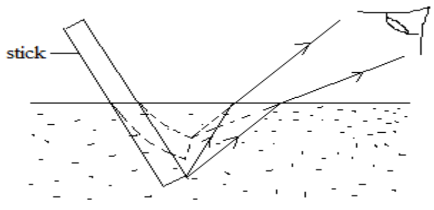
If light travels from a less dense medium to a denser medium, it is refracted towards the normal at the point of incidence. However, if light travels from a denser medium to a less dense medium, it is refracted away from the normal at the point of incidence.





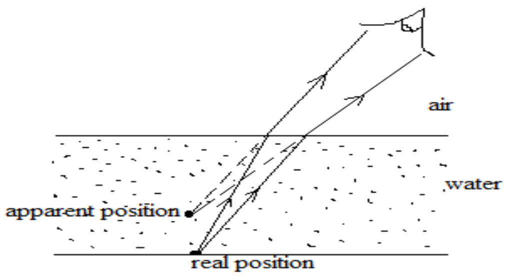
Explanation of some of the effects of refraction:

1. A stick partially immersed in water appears bent.



Any stick held in a slanting position in water appears to be bent. This is because when light rays passes from an object under water to air i.e. from a dense medium to a less dense medium, they are refracted away from the normal. As a result, part of the stick under water appears raised up. The part outside is seen in its original position and the end result is that the stick appears bent.

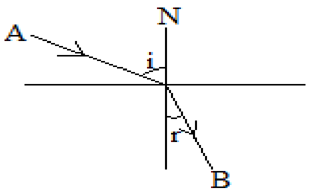
2. A pond appears shallower than its actual depth.



Light rays proceeding from an object at the bottom of the pond travels from water to air. After reaching the water-air interface, they are refracted away from the normal to the observer's eye. To an observer, the object (at the bottom of the pond) appears to be in the same straight line with the rays entering the observer's eye which makes it appear to be raised and the pond appears shallower than its really is.

LAWS OF REFRACTION:

Consider a ray of light incident on an interface between two media as shown below.



$i = angle\ of\ incidence, r = amgle\ of\ refraction, O = point\ of\ incidence, ON = normal\ at\ point, O, AO = the\ incident\ ray, OB = refracted\ reay.$

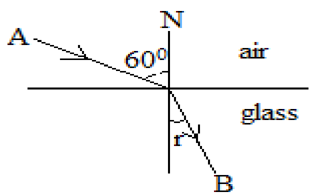
Law 1. The incident ray, the refracted ray and the normal at the point of incidence all lie in the same plane.

Law 2. The ratio of sine of angle of incidence to sine of angle refraction is a constant for a given pair of media. This is called Snell's law. The constant ratio is called the refractive index.

$\frac{\sin i}{\sin r} = constant, \rightarrow \frac{\sin i}{\sin r} = n$

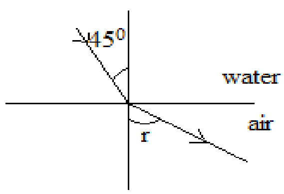
Examples:

1.



Find the angle of refraction if the refractive index of glass is 1.52.
Solution:
$n = \frac{\sin i}{\sin r} \rightarrow 1.52 = \frac{\sin 60}{\sin r} \rightarrow r = \sin^{-1}\left(\frac{\sin 60}{1.52}\right) \therefore r = 34.7^0$

2. The angle of incidence in water of refractive index 1.33 is 45^0. Find the angle of refraction.
Solution:



$n_w \sin 45 = n_a \sin r \rightarrow r = \sin(1.33 \sin 45) = 70.13^0$

REFRACTIVE INDEX

Is defined as the ratio of sine of angle of incidence to sine of angle of refraction for a given pair of media of different optical densities.

This refractive index is also known as the relative refractive index.

Note: if light is incident from air or vacuum of any other medium, then the refractive index is called the absolute refractive index of the medium.

Absolute refractive index.

This is the ratio of sine of angle of incidence to sine of angle of refraction for a ray of light travelling from a vacuum (air) to a given medium.

Absolute refractive index of a given medium is also known as the refractive index of a material.

According to the theory of light, refractive index can also be defined as the ratio of speed of light in one medium to the speed of light in another medium of different optical densities.

$${}_1n_2 = \frac{speed\ of\ light\ in\ medium\ 1}{speed\ of\ light\ in\ medium\ 2}$$

$${}_1n_2 = \frac{V_1}{V_2}$$

For absolute refractive index of a given medium, it is defined as the ratio of the speed of light in a vacuum to the speed of light in a given medium.

Refractive index= $\frac{speed\ of\ light\ in\ a\ vacuum,\ c}{speed\ of\ light\ in\ a\ medium,\ v}$ $\rightarrow n = \frac{c}{v}$

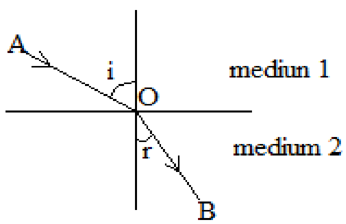
Where the speed of light in a vacuum, $c = 3.0 \times 10^8 ms^{-1}$

Note: refractive index of a vacuum or air is 1.

THE PRINCIPLE OF REVERSIBILITY OF LIGHT.

It states that when the object and image positions are interchanged, light rays will always trace the same path.

This means that a ray of light can travel from medium 1 to 2 and from 2 to 1 along the same path if the position of the object is interchanged.



$${}_1n_2 = \frac{\sin i}{\sin r} \text{ .... (i)}$$

If the light source is altered so that it moves from B to O, then applying Snell's law,

$${}_2n_1 = \frac{\sin r}{\sin i} \text{ .... (ii)}$$

$$\rightarrow {}_2n_1 = \frac{1}{\frac{\sin i}{\sin r}}$$

$$\boxed{{}_2n_1 = \frac{1}{{}_1n_2}}$$

Example.

If the refractive index ${}_{air}n_{water} = \frac{4}{3}$. Find the ${}_{water}n_{air}$.

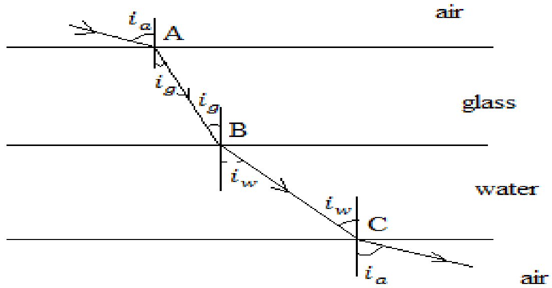
Solution:

$_an_w = \frac{4}{3}$ But $_wn_a = \frac{1}{_an_w} \rightarrow {_wn_a} = \frac{1}{\frac{4}{3}} \therefore {_wn_a} = \frac{3}{4}$

REFRACTION AT PARALLEL PLENE BOUNDARIES

Consider a ray of light travelling from air to glass to water and back to air as shown in the illustration below.

Experiment to show that if the boundaries in between the medium are parallel, then the emergent ray is parallel to the incident ray and the angle of incidence is equal to the emergent angle.



Applying Snell's law at A,

$_an_g = \frac{\sin i_a}{\sin i_g}$ …………………. (i)

Applying Snell's law at B,

$_gn_w = \frac{\sin i_g}{\sin i_w}$ …………………. (ii)

Applying Snell's law at C,

$_wn_a = \frac{\sin i_w}{\sin i_a}$ …………………. (iii)

Multiplying (i), (ii) and (iii).

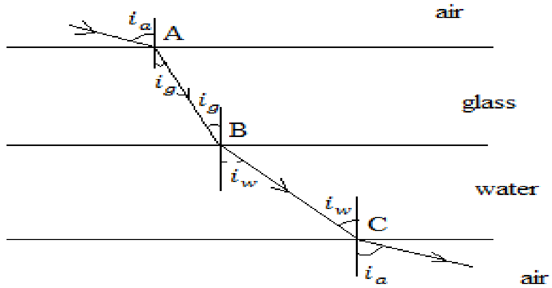
$_an_g \cdot {_gn_w} \cdot {_wn_a} = \frac{\sin i_a}{\sin i_g} \cdot \frac{\sin i_g}{\sin i_w} \cdot \frac{\sin i_w}{\sin i_a} \quad \rightarrow {_an_g} \cdot {_gn_w} \cdot {_wn_a} = 1$

But $_wn_a = \frac{1}{_an_w}$

$_an_g \cdot {_gn_w} \cdot \frac{1}{_an_w} = 1 \quad \therefore \boxed{{_an_g} \cdot {_gn_w} = {_an_w}}$

Relationship between refractive index, n andsin i ($n \sin i = constant$)

Consider a ray of light moving from air to glass and then back to air as shown below.



Applying Snell's law at A,

$_an_g = \frac{\sin i_a}{\sin i_g}$ But $_an_g = \frac{n_g}{n_a} \rightarrow \frac{n_g}{n_a} = \frac{\sin i_a}{\sin i_g} \therefore n_a \sin i_a = n_g \sin i_g$ ……………… (i)

Applying Snell's law at B,

$_gn_w = \frac{\sin i_g}{\sin i_w}$. But $_gn_w = \frac{n_w}{n_g} \rightarrow \frac{n_w}{n_g} = \frac{\sin i_g}{\sin i_w} \therefore n_g \sin i_g = n_w \sin i_w$ ……………… (ii)

But from (i), $n_g \sin i_g = n_a \sin i_a$

Therefore $n_a \sin i_a = n_g \sin i_g = n_w \sin i_w$

In general $\boxed{n \sin i = constant}$

Examples.

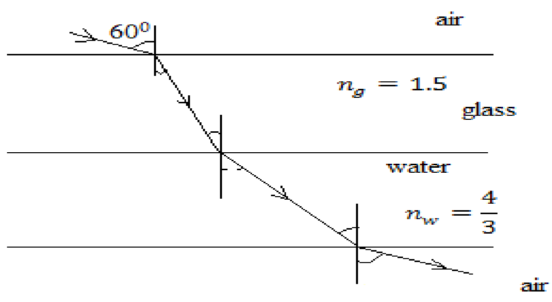
1. The refractive index of water is $\frac{4}{3}$ and that of a plane glass is $\frac{3}{2}$. calculate the refractive index for;
   (i) Light travelling from water to glass
   (ii) Light travelling from glass to water.

   Solution:

   (i) $_wn_g = \frac{n_g}{n_w} \rightarrow {_wn_g} = \frac{3}{2} \times \frac{3}{4} = \frac{9}{8}$

   (ii) $_gn_w = \frac{n_w}{n_g} = \frac{4}{3} \times \frac{2}{3} = \frac{8}{9}$

2. In the figure below, light is incident at an angle of 60^0 on a medium of parallel boundary.



Find;

(i) Angle of refraction in glass

(ii) Angle of incidence at the water, air interface

(iii) Refractive index for light travelling from glass to water

(iv) Refractive index for light travelling from glass to air.

Solution:

(i) $n \sin i = constant$

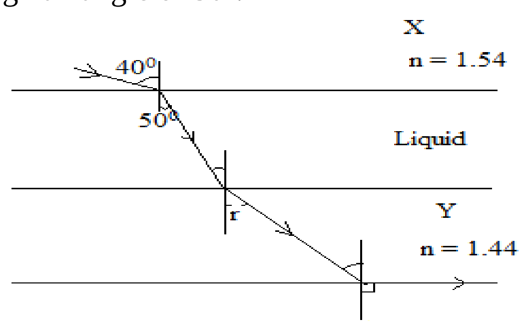
$n_a \sin i_a = n_g \sin i_g \rightarrow \sin 60 = 1.5 \sin r \therefore r = \sin^{-1}(\frac{\sin 60}{1.5}) = 35.26^0$

(ii) $n_w \sin i_w = n_a \sin i_a \rightarrow \frac{4}{3} \sin i_w = \sin 60 \therefore i_w = \sin^{-1}(\frac{3}{4} \sin 60) = 40.5^0$

(iii) ${}_g n_w = \frac{n_w}{n_g} = \left(\frac{4}{3}\right) \div 1.5 = \frac{8}{9} = 0.89$

(iv) ${}_g n_a = \frac{n_a}{n_g} = \frac{1}{1.5} = \frac{2}{3}$

3. The figure below shows a layer of liquid confined between two transparent plates x and y of refractive index 1.54 and 1.44 respectively. A ray of monochromatic light makes an angle of 40^0 with the normal to the interface between x and the liquid. The ray is the refracted through an angle of 50^0.



Find

(i) Refractive index of the liquid

(ii) The angle of refraction in medium Y

(iii) Minimum angle of incidence in X for which the ray of light will not emerge from medium Y.

Solution:

(i) $n \sin i = constant$

$n_x \sin i_x = n_l \sin i_l, \quad \therefore\ 1.54 \sin 40 = n_l \sin 50 \ \rightarrow n_l = 1.2922$

(ii) $n \sin i = constant$

$n_l \sin i_l = n_y \sin i_y, \quad \therefore\ 1.2922 \sin 50 = 1.44 \sin r \ \rightarrow r = 43.43^0$

(iii) Applying Snell's law at C.

$n_a \sin i_a = n_y \sin i_y, \quad \therefore\ 1 \sin 90 = 1.44 \sin r_c \ \rightarrow r_c = 43.98^0$

Applying Snell's law at B.

$n_y \sin i_y = n_l \sin i_l, \quad \therefore\ 1.44 \sin 43.98 = 1.2922 \sin r_B \ \rightarrow r_B = 50.7^0$

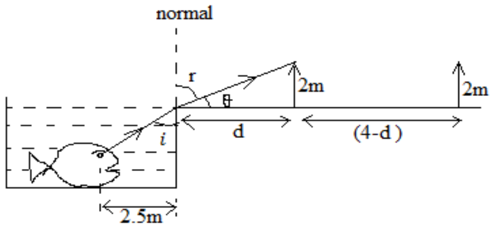
Applying Snell's law at A.

$n_L \sin i_L = n_X \sin i_X, \quad \therefore\ 1.54 \sin i = 1.2922 \sin 50.7 \ \rightarrow i = 40.5^0$

4. A small fish is 3.0m below the surface of a pond and 2.5m from the bank. A man 2.0m tall stands 4.0m from the pond. Assuming that the side of the pond are vertical, calculate

the distance the man should move towards the edge of the pond before visible becomes visible to the fish. (Refractive index of water = 1.33).

Solution:



From the diagram $\tan i = \frac{2.5}{3} \rightarrow i = 39.81^0$

Applying Snell's law at the edge of the pond gives

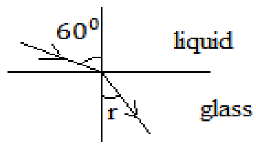
$n_w \sin i_w = n_a \sin i_a \rightarrow 1.33 \sin 39.81 = 1 \sin r \rightarrow r = 58.3^0$

Therefore $\theta = 90^0 - 58.4^0 = 31.6^0$

From the diagram $\tan \theta = \frac{2}{d} \rightarrow d = \frac{2}{\tan 31.6} \therefore d = 3.2m$

The required distance travelled $= 4 - d = 4 - 3.2 = 0.8m$

5. A monochromatic light is incident from a liquid onto the upper surface of a transparent glass block as shown below.



Given that the speed of light in the liquid is $2.4 \times 10^8 m^{-s}$ and that in glass is $1.29 \times 10^8 m^{-s}$. Find the angle of refraction, r.

Solution:

Applying Snell's law,

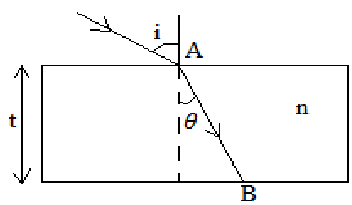
$n_l \sin 60 = n_g \sin r$ But $n_l = \frac{c}{V_l}$, and $n_g = \frac{c}{V_g}$

$\frac{c}{V_l} \sin 60 = \frac{c}{V_g} \sin r \quad \rightarrow \frac{3\times10^8}{2.4\times10^8} \sin 60 = \frac{3\times10^8}{1.29\times10^8} \sin r$

$r = 27.74^0$

6. A monochromatic light incident on a block of material placed in a vacuum is refracted through an angle θ . If the block has a refractive index n and is of thickness, t. show that light takes time $\frac{nt \sec \theta}{c}$.

Solution:

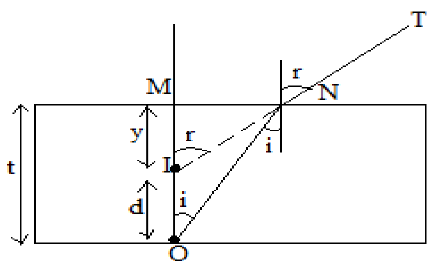


$time = \frac{distance}{speed}$

$$\cos\theta = \frac{t}{AB} \rightarrow AB = \frac{t}{\cos\theta} \rightarrow AB = t\sec\theta \;\; \therefore distance = t\sec\theta$$
$$n = \frac{c}{v} \rightarrow v = \frac{c}{n} \;\; \therefore speed = \frac{c}{n}$$
$$time = t\sec\theta \div \frac{c}{n} = t\sec\theta \times \frac{n}{c}$$
$$\therefore time = \frac{nt\sec\theta}{c}$$

REAL AND APPARENT DEPTH.

Consider an object O at a distance, t below the surface of a medium such as water or glass.



Applying Snell's law at N,

$n_g \sin i = n_a \sin r \rightarrow n_g = \frac{\sin r}{\sin i}$ ……………… (i)

But $\sin i = \frac{MN}{ON}$ ……………….. (ii)

And $\sin r = \frac{MN}{IN}$ ……………….. (iii)

Substituting (ii) and (iii) into (i) gives

$$n_g = \frac{MN}{IN} \div \frac{MN}{ON}$$

$$n_g = \frac{ON}{IN}$$

But N is very close to M such that $ON \approx OM$ and $IN \approx IM$

$$n_g = \frac{OM}{IM} \therefore n = \frac{real\ depth}{apparent\ depth}$$

$$n = \frac{t}{apparent\ depth} \rightarrow apparent\ depth = \frac{t}{n}$$

But apparent displacement $d = t - \frac{t}{n}$

$\therefore \boxed{d = t(1 - \frac{1}{n})}$ Which is the apparent displacement (vertical displacement)

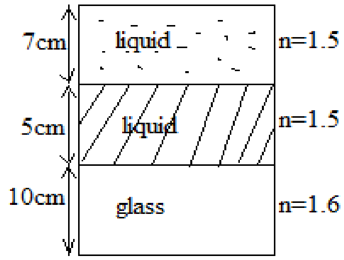
Note: When the object is viewed through several media whose boundaries are parallel, then the total displacement is equal to the sum of the displacement produced by each medium.

QN: Find the expression for the distance through which an object appears to be displaced towards the eye when a plate of glass of thickness t and refractive index, n is used.

Example:

1. A container contains a slab of thickness 10cm and refractive index 1.6. The slab contains a liquid of thickness 5cm and refractive index 1.5. Above this liquid, floats another liquid of thickness 7cm and refractive index 1.5. Determine the displacement of a point at the bottom of the tank.

   Solution:



$d = t(1 - \frac{1}{n})$

$d = t_g\left(1 - \frac{1}{n_g}\right) + t_l(1 - \frac{1}{n_l}) + t_l(1 - \frac{1}{n_l})$

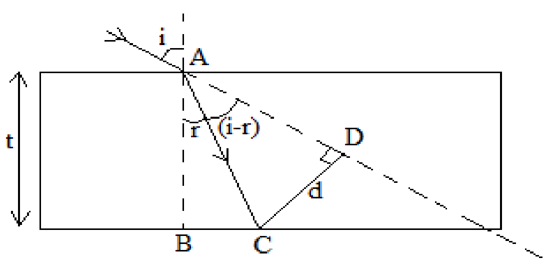
$d = 10\left(1 - \frac{1}{1.6}\right) + 5(1 - \frac{1}{1.5}) + 7(1 - \frac{1}{1.5})$

$d = 7.75cm$

SIDE WAYS (LATERAL) DISPLACEMENT.

When light travels from one medium to another, its direction is displaced sideways. This is called lateral displacement.

Consider a ray of light incident at an angle i on the upper surface of a glass block of thickness, t and then suddenly refracted through an angle, r causing it to suffer a sideways displacement, d.



Consider triangle ACD

$\sin(i - r) = \frac{d}{AC} \rightarrow d = AC \sin(i - r)$ …………………. (i)

Consider triangle ABC

$\cos r = \frac{t}{AC} \rightarrow AC = \frac{t}{\cos r}$ ……………………. (ii)

Substituting (ii) into (i)

$d = \left(\frac{t}{\cos r}\right)\sin(i-r)$ $\therefore$ $\boxed{d = \frac{t\sin(i-r)}{\cos r}}$ OR $\boxed{d = t\sec r\sin(i-r)}$

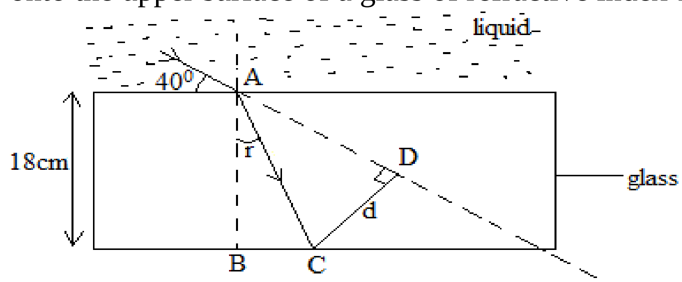
Horizontal distance, BC

From $\tan r = \frac{BC}{t}$ $\rightarrow BC = t(\tan r)$

$\boxed{BC = t(\tan r)}$

Examples:

1. The figure below shows a monochromatic ray of light incident from a liquid of refractive index 1.33 onto the upper surface of a glass of refractive index 1.42.

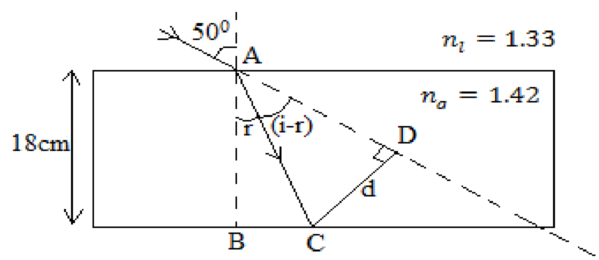


Calculate;

(i) Horizontal displacement, AB

(ii) Lateral displacement BC of the emergent light.

Solution.



(i) Applying Snell's law at the liquid-glass interface gives

$n_l \sin i_l = n_g \sin i_g$ $\rightarrow$ $1.33 \sin 50 = 1.42 \sin r$ $\therefore r = 45.8^0$

Horizontal displacement $BC = t\tan r$

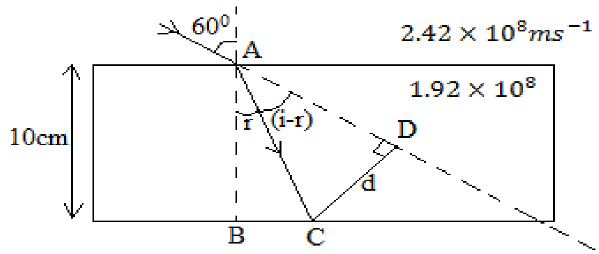
$BC = 18\tan 45.8^0 = 18.51cm$

(ii) Lateral displacement $d = \frac{t\sin(i-r)}{\cos r}$

$d = \frac{18\sin(50-45.8)}{\cos 45.8} = 1.89cm$

2. A monochromatic light is incident from a liquid onto the upper surface of a transparent glass block whose sides are parallel. If the speed of light in the liquid is $2.42 \times 10^8 ms^{-1}$ and the speed of light in glass is $1.92 \times 10^8 ms^{-1}$. Calculate the lateral displacement of the emergent ray. (Angle of incident in the liquid is 60^0 and the thickness, t is 10cm).

Solution.



$n_l = \frac{3.0\times10^8}{2.42\times10^8} = 1.24$ And $n_g = \frac{3.0\times10^8}{1.92\times10^8} = 1.56$

Applying Snell's law at the liquid-glass interface gives

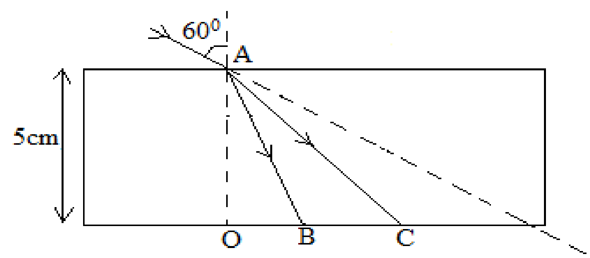
$n_l \sin i_l = n_g \sin i_g \rightarrow 1.24 \sin 60 = 1.56 \sin r \quad \therefore r = 43.5^0$

Lateral displacement $d = \frac{t \sin(i-r)}{\cos r}$

$d = \frac{10 \sin(60-43.5)}{\cos 43.5} = 3.92cm$

3. Light consisting of red and blue light is incident on air – glass interface. The two colors emerge from the glass block at two point P and Q respectively. If the speed of blue and red light in glass are $1.88 \times 10^8 ms^{-1}$ and $1.94 \times 10^8 ms^{-1}$ respectively. Calculate the distance PQ. (Angle of incidence in air = 60^0 and thickness of the glass = 5cm).

Solution:



$n = \frac{speed\ of\ light\ in\ a\ vacuum}{speed\ of\ light\ in\ a\ medium} = \frac{c}{v}$

For blue light,

$n_B = \frac{3.0\times10^8}{1.88\times10^8} = 1.596$

For red light,

$n_R = \frac{3.0\times10^8}{1.94\times10^8} = 1.546$

$n \sin i = constant$

$1 \times \sin 60 = 1.546 \sin r_R \rightarrow r_R = 34.1^0$

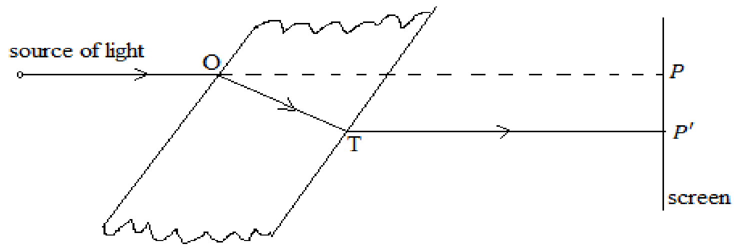
$1 \times \sin 60 = 1.596 \sin r_B \rightarrow r_B = 32.9^0$

$\tan r_R = \frac{OQ}{5} \rightarrow OQ = 5 \tan 34.1 = 3.39cm$

$\tan r_B = \frac{OP}{5} \rightarrow OP = 5 \tan 32.9 = 3.23cm$

$PQ = 3.39 - 3.32 = 0.16cm$

4. A monochromatic source of light in air sends a narrow beam of light perpendicular to a screen 2.0m away. The beam strikes the screen at P. A glass block of refractive index 1.5 and thickness 10cm is inserted as shown so that the beam strikes it at an angle of 30^0.



Find;

(i) The angle of refraction at the first surface

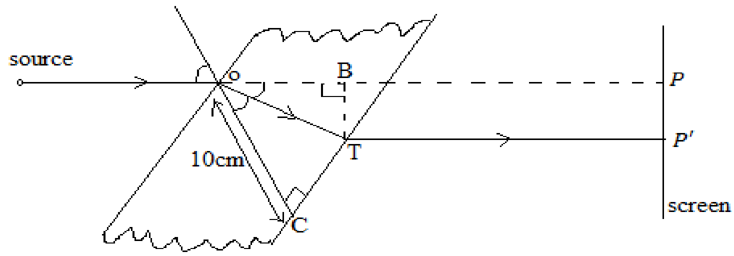
(ii) The distance OT

(iii) The speed of the beam through glass block

(iv) Time taken to cover distance OT

(v) Lateral displacement of the beam.

Solution.



(i) $n \sin i = constant$

$1 \times \sin 30 = 1.5 \times \sin r \quad \rightarrow r = 19.47^0$

(ii) ΔOCT: $\cos 19.47 = \frac{0.1}{OT} \quad \rightarrow OT = 0.11m$

(iii) $n = \frac{speed\ of\ light\ in\ a\ vacuum}{speed\ of\ light\ in\ a\ medium} = \frac{c}{v}$

$v_g = \frac{3.0\times10^8}{1.5} = 2.0 \times 10^8 ms^{-1}$

(iv) $time = \frac{distance}{speed} = \frac{0.11}{2.0\times10^8} = 5.5 \times 10^{-10} s$

(v) Lateral displacement $d = \frac{t \sin(i-r)}{\cos r}$

$d = \frac{0.1 \sin(30-19.5)}{\cos 19.5} = 0.0193m$

5. A microscope is focused on a mark on a white paper. When the mark is covered by a plate of glass 2cm thick, the microscope has to be raised by o.67cm for the mark to be once more in focus. Calculate the refractive index of the glass.

Solution.

From $d = t(1 - \frac{1}{n})$

$0.67 = 2\left(1 - \frac{1}{n}\right) \quad \therefore n = 1.5$

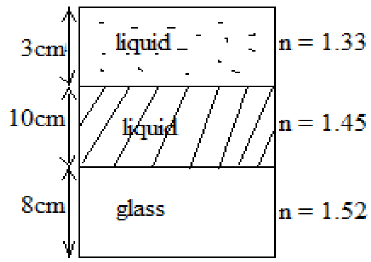
6. An object is 6cm below the tank of water of refractive index 1.33. Determine the displacement of the object to an observer directly above the tank.

Solution.

From $d = t(1 - \frac{1}{n}) \rightarrow d = 6\left(1 - \frac{1}{1.33}\right) = 1.4887cm$

7. A tank contains a slab of glass 8cm thick and of refractive index 1.52. Above this is a liquid 10cm thick and of refractive index 1.45 and floating on it is 3cm of water of refractive index 1.33. Find the apparent position of the mark below the tank.

Solution:



$d = t(1 - \frac{1}{n})$

$d = t_g\left(1 - \frac{1}{n_g}\right) + t_l(1 - \frac{1}{n_l}) + t_l(1 - \frac{1}{n_l})$

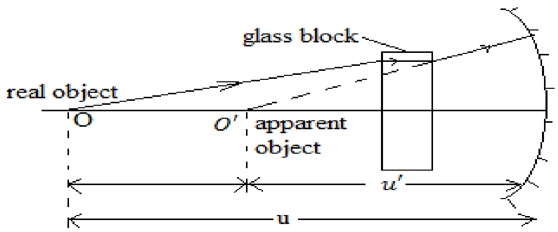
$d = 8\left(1 - \frac{1}{1.52}\right) + 10(1 - \frac{1}{1.45}) + 3(1 - \frac{1}{1.33})$

$d = 6.5847cm$

Apparent position = (3+10+8) – 6.5847 = 14.4153cm

8. A small object is placed 20cm in front of a concave mirror of focal length 15cm. a parallel sided glass block of thickness 6cm and refractive index 1.5 is the placed between the mirror and the object. Find the shift in the position of the image.

Solution.



Consider the action of a concave mirror in the absence of a glass block

$u = 20cm, f = 15cm$

$\frac{1}{u} + \frac{1}{v} = \frac{1}{f} \rightarrow \frac{1}{20} + \frac{1}{v} = \frac{1}{15} \therefore v = 60cm.$

Thus in the absence of a glass block, **image distance** = 60cm

In this case, **magnification**, $m = \frac{v}{u} = \frac{60}{20} = 3$

Consider the action of a glass block

Using the relation $d = t\left(1 - \frac{1}{n}\right)$

$d = 6\left(1 - \frac{1}{1.5}\right) = 2cm$

Thus in the presence of a glass block, the object distance $u' = (20 - 2) = 18cm$

(The object is displaced and appears to be 18cm in front of the mirror)

Consider the action of a concave mirror in the presence of a glass block

$u' = 18cm, f = 15cm$

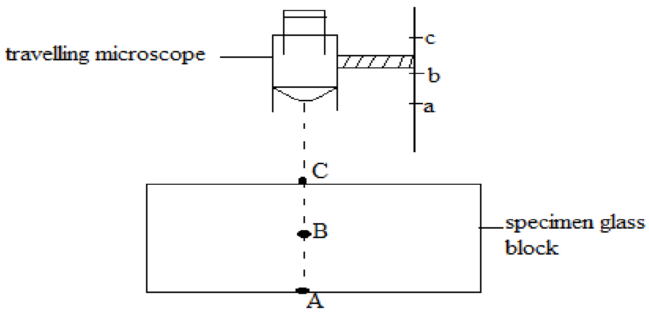
$\frac{1}{u'} + \frac{1}{v'} = \frac{1}{f} \rightarrow \frac{1}{18} + \frac{1}{v'} = \frac{1}{15} \therefore v' = 90cm.$

The required shift in the image position= $v' - v = 90 - 60 = 30cm$

The magnification now becomes $m = \frac{v'}{u'} = \frac{90}{18} = 5$

DETERMINATION OF REFRACTIVE INDEX BY REAL AND APPARENT DEPTH METHOD.

QN: Describe how you would determine the refractive index of a material inform of a glass block.



A cross is made on a white sheet of paper and the paper is placed under a travelling microscope.

The microscope is adjusted until the cross is focused clearly. The scale reading of is recorded as, a.

The test glass block is placed on the cross and the microscope is again adjusted until the cross is clearly seen. The scale reading is recorded as, b.

Lycopodiun powder (chalk dust) is now sprinkled on top of the glass block.

The microscope is again adjusted until the particles are seen clearly. The scale reading is recorded as,

The refractive index n is calculated from

$n = \frac{c-a}{c-b}$

Examples.

1. A microscope is focused on a scratched at the bottom of an empty glass dish. Water is then poured in and it is found that the microscope has to be raised by 1.2cm for refocusing. Chalk dust is sprinkled on the surface of water and the dust comes into focus when the microscope is raised by an additional 3.5cm. Find the refractive index of water.
   Solution.
   $n = \frac{real\ depth}{apparent\ depth} = \frac{c-a}{c-b} = \frac{(3.5+1.2)-0}{(3.5+1.2)-1.2} = \frac{4.7}{3.5} = 1.34$

Exercise

1. Light of two colors, blue and red is incident at an angle α from air to a glass block of thickness t. when blue and red light are refracted through angles of θ_b and θ_r respectively, their corresponding speeds in the glass block are V_b and V_r . Show that the separation of the two colors at the bottom of the glass block$d = \frac{t}{c}\left(\frac{V_r}{\cos\theta_r} - \frac{V_b}{\cos\theta_b}\right)\sin\alpha$. Where $\theta_r > \theta_b$ and c is the speed of light in air.