ELECTROMAGNETISM AND ELECTROMAGNETIC INDUCTION

1. Electromagnetism.

This is a fundamental interaction between an electric field and magnetic field.

Magnetism in matter

Magnetism refers to a property of a substance (or a material) to attract and hold other substances (or materials).

NB:
A material that exhibits magnetism is known to be a **magnetic material**.

Materials affected by magnets are classified into; -
1. Ferro-magnetic materials.
2. Diamagnetic materials.
3. Paramagnetic materials.

1. Ferro-magnetic materials.

These refer to materials which are strongly attracted by a magnet.

Examples of Ferro-magnetic materials include; -

* Iron.
* Steel.
* Cobalt.
* Nickel.
* Alloys such as perm-alloy and alnico, etc.

2. Diamagnetic materials.

These refer to materials that are slightly (weakly) repelled by a strong magnet.

Examples of diamagnetic materials include; -

* Zinc.
* Bismuth.
* Benzene.
* Sodium chloride.
* Gold.
* Mercury, etc.

Note:

Diamagnetic materials when placed in a magnetic field, they are magnetised in the direction opposite to the magnetising field.

3. Paramagnetic materials.

These refer to materials which are slightly (weakly) attracted by a strong magnet.

Examples of paramagnetic materials include; -

* Wood.
* Aluminium.
* Uranium.
* Platinum.
* Oxygen.
* Copper (II) sulphate, etc.

NB

(i) Paramagnetic materials become more magnetic when they are very cold.

(ii) Since paramagnetic materials are slightly attracted by a strong magnet, then they are considered to be non-magnetic materials.

Note

1. The magnetic materials made from powders of iron oxide and barium oxide are known as **Ferrites**.

   An example of a very strong magnet made from ferrites is Ceramic magnet (Magnadur magnet)

2. Other strong magnets are made from alloys of iron, nickel, copper, cobalt and aluminium.

Properties of a magnet.

(i) When a bar magnet is freely suspended, it always comes to rest while pointing in North-South direction, i.e., *its north pole points in the earth's geographical north and south pole in the earth's geographical south.*

(ii) The magnetic force is strongest (more concentrated) at the poles than the centre of the magnet.

(iii) Like poles of a magnet repel each other, unlike poles of a magnet attract each other.

NB:

(i) The third property of magnets is known as the **"law of magnetism" (or the first law of magnetism)**.

(ii) There are 3 major ways of making a material to become a magnet, namely;

* *Electrical method.*
* *Stroking (Touch) method.*
* *Absolute method.*

<u>Magnetic properties of Steel and Iron (Hard and Soft magnetic materials)</u>

Hard magnetic materials:

These refer to Ferro-magnetic materials which take long to be magnetized and can retain their magnetism for a long time after the external magnetic field is removed.

Example of a hard magnetic material is **<u>hard steel</u>**.

2. **Soft magnetic materials:**

These refer to Ferro-magnetic materials which can easily be magnetized but can not retain their magnetism after the external magnetic field is removed.

Example of a soft magnetic material is **<u>iron</u>**.

NB:

(a) A magnet made from a hard magnetic material is said to be a **permanent magnet**, while a magnet made from a soft magnetic material is said to be a **temporary magnet**.

(b) Recent special alloys for making **powerful permanent** magnets have trade names such as; -

* Alcomax

* Alnico
* Ticonal.

They contain iron, nickel, cobalt aluminium and copper in various proportions.

(c) Recent special alloys for making **temporary** magnets include; -

* Mumental (nickel + iron + copper)
* Stalloy (iron + silicon)

NB:

(a) **The use of a hard magnetic material is** for making permanent magnets used in loudspeakers, dynamos, telephone earpiece, etc.

(b) **Soft magnetic materials are used; -**

* for making electromagnets.
* as magnetic keepers, for proper storage of magnets.

Magnetic Field and magnetic field lines

Magnetic field

This is a region around a magnet where the magnetic force is experienced (felt).

A magnetic field is a vector quantity and it can be graphically represented by **<u>magnetic field lines</u>** which indicate its strength and direction.

NB

1. **The direction** of the <u>magnetic field</u> at any given point refers to the direction of the force on a north pole at that point.

The direction of the magnetic force is represented by the magnetic field lines, also known as lines of magnetic force or lines of magnetic flux.

Magnetic field line or ***line of magnetic force*** or ***line of magnetic flux*** refers to the path which shows the direction that a north pole would follow when placed in a magnetic field.

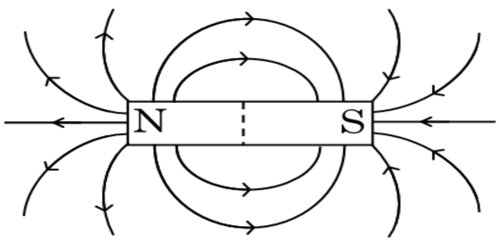
2. The magnetic field lines can be thought of as closed loops with one part inside the magnet and the other part outside the magnet.

Properties (or Characteristics) of magnetic field lines.

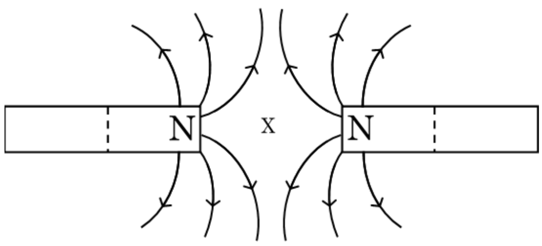
* Outside the magnet, the magnetic field lines start from the North pole and end on the South pole. Inside the magnet, the field lines continue from South pole to North pole.
* Magnetic field lines never cross each other.
* The magnetic field lines are close to each other where the magnetic field is strongest (at the poles) and further apart where the field is weak (middle of the magnet).

Magnetic field pattern due to magnets

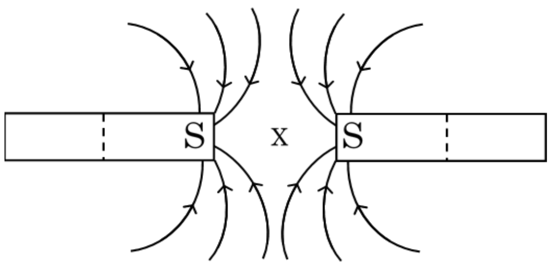
1. Single isolated bar magnet



2. (a) Two north poles near each other



(b) Two south poles near each other.



NB:

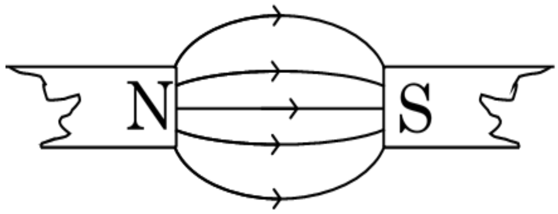
The field around a bar magnet is non uniform, i.e. the strength and direction vary from one place to another.

Effect of magnetic material on magnetic field lines

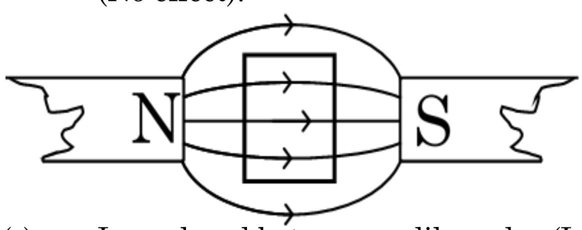
* Non-magnetic materials like copper, have no effect on magnetic field lines.
* Magnetic materials such as **iron** concentrate magnetic field lines.

Illustration

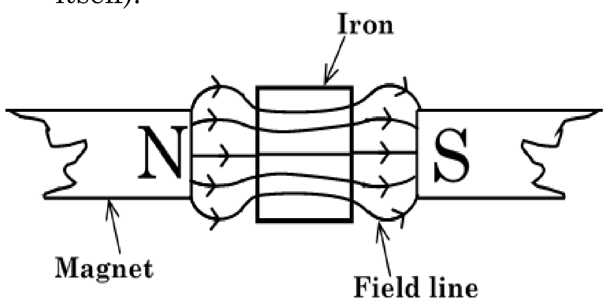
(a) No material placed between unlike poles (No effect).



(b) Copper placed between unlike poles (No effect).



(c) Iron placed between unlike poles (Iron concentrates the field lines within itself).



THE EARTH AS A MAGNET

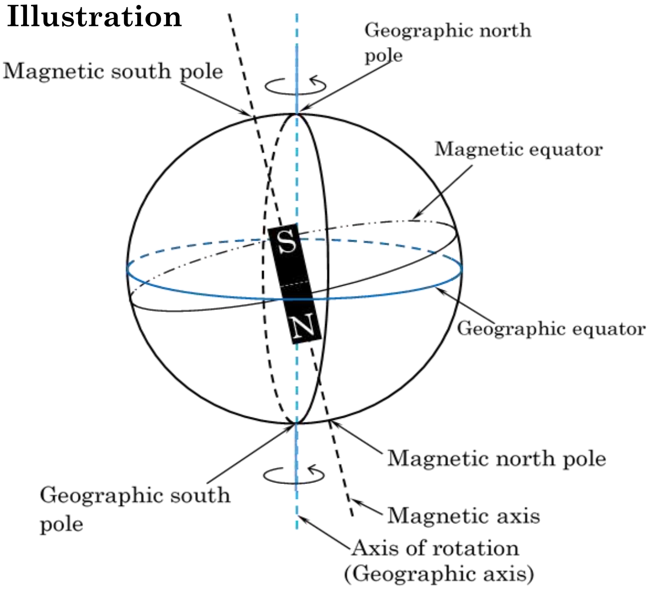
✓ The Earth consists of a **solid iron core** which is surrounded by an ocean of **hot, liquid metal**.
✓ The liquid metal flows in Earth's core and creates electrical currents, which

in turn create magnetic field within and around the earth.

- ✓ This field affects a compass needle as one moves on earth's surface from one point to another.
- ✓ Thus, the earth behaves as though it contains a short bar magnet inside it inclined at a small angle to its axis of rotation.
- ✓ The earth's magnetic north-pole is conventionally in the southern hemisphere and its magnetic south-pole in the northern hemisphere.

Therefore, the south pole of an imaginary earth's magnet attracts the north pole of the suspended bar magnet or the north pole of the compass needle

Illustration



The Earth

Northern hemisphere

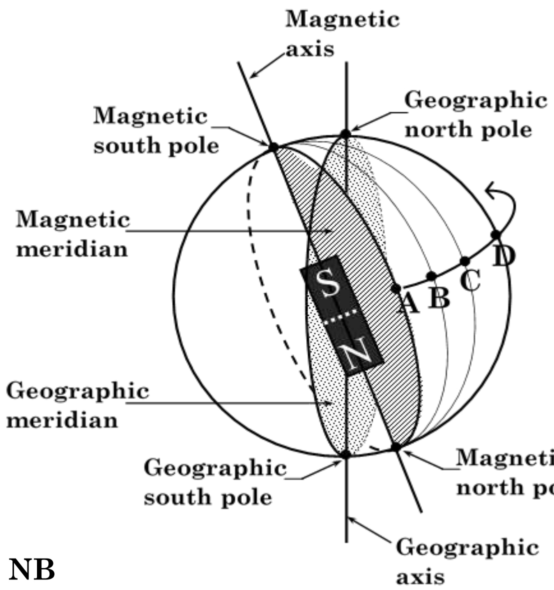
Southern hemisphere

Elements of the earth's magnet.

There are three major elements of the earth's magnet

(i) Magnetic meridian
(ii) Geographic meridian
(iii) Angle of declination (Magnetic variation)

Illustration



NB

A, B, C and **D** are different positions on the earth's surface lying on different magnetic meridians as a person rotates along the earth's surface in an anti-clockwise direction.

Definitions

(i) Magnetic meridian

This refers to *the vertical plane containing the magnetic axis of a freely suspended magnet at rest under the action of the earth's magnetic field*. **OR,**

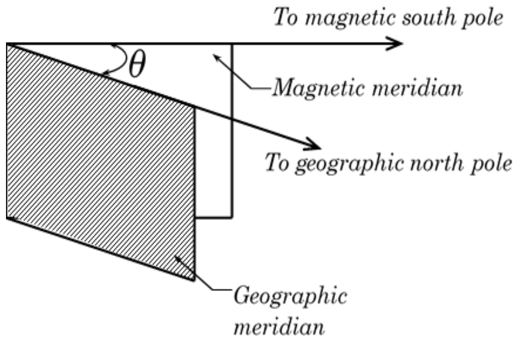
It is the *vertical plane passing through the earth's magnetic south and north poles.*

(ii) Geographic meridian

This refers to *the vertical plane which passes through the earth's geographic north and south poles.*

(iii) Magnetic variation (Angle of declination)

This refers to *the angle in the horizontal plane between the earth's magnetic and geographic meridian.*



Note

(i) The angle of declination changes depending on where an observer is positioned on the earth's surface. At a particular place, the magnetic variation can change with time due to changing position of the earth's magnetic polarities.

(ii) When the magnetic axis and the geographic axis are in line as seen by the observer, then the angle of declination is zero.

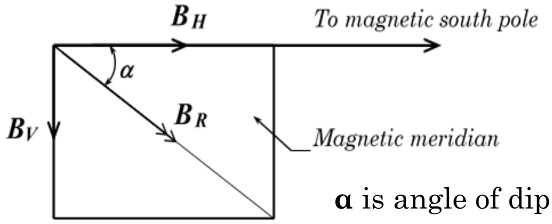
(iii) Magnetic axis refers to *the imaginary line passing through the earth's magnetic north and south poles.*

(iv) Geographic axis refers to *the imaginary line through the center of the earth and passing through the geographical north and south poles.*

Angle of inclination (angle of dip)

This refers *to the angle between the horizontal surface of the earth and the direction of the earth's magnetic field at a particular point on the earth's surface.* **OR,**

It is *the angle between the horizontal and the magnetic axis of a freely suspended magnet.*



α is angle of dip.

B_V is the vertical component of the earth's magnetic field.

B_H is the horizontal component of the earth's magnetic field.

B_R is the resultant earth's magnetic field.

Note

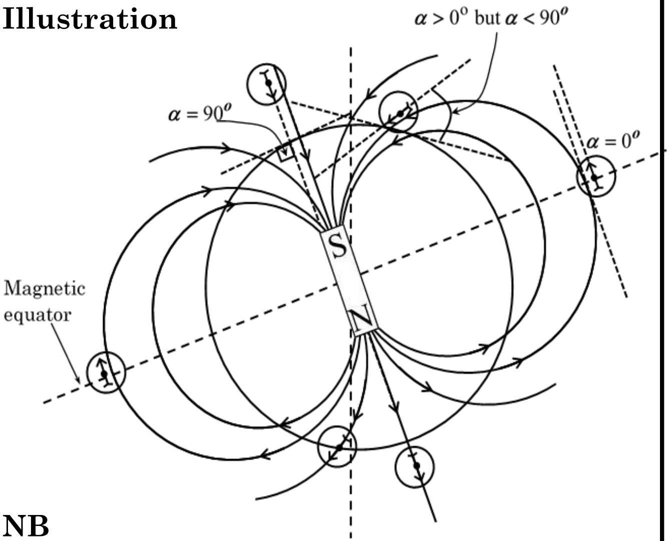
(i) Angle of dip can be calculated from,

$$\alpha = Tan^{-1}\left(\frac{B_V}{B_H}\right)$$, if B_H and B_V are known.

(ii) In the northern hemisphere, the north pole of the compass needle dips and in the southern hemisphere, the south pole of the compass needle dips.

(iii) As an observer moves from the magnetic equator towards the magnetic south pole, the angle of dip keeps changing. At the magnetic equator, the earth's magnetic field lines are parallel to the horizontal (earth's surface); therefore, the angle of dip at the magnetic equator is zero degrees (0^o). As the observer moves along a given longitude towards the geographic north pole (or magnetic south pole), the resultant magnetic field lines meet the earth's surface at angles greater than 0^o but less than 90^o, thus the angle of dip at such a position is also greater than 0^o but less than 90^o. At the earth's magnetic south pole, the magnetic field lines are normal to the surface of the earth, thus they are perpendicular to the horizontal. Therefore, the angle of dip is 90^o.

Illustration



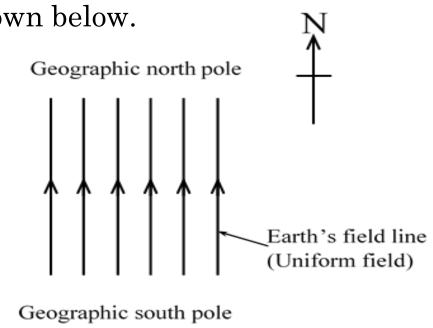
NB

(i) The symbol is a compass needle.

(ii) Magnetic equator refers to the greatest circle in a horizontal plane perpendicular to the magnetic meridian where a freely suspended bar magnet experiences zero magnetic dip.

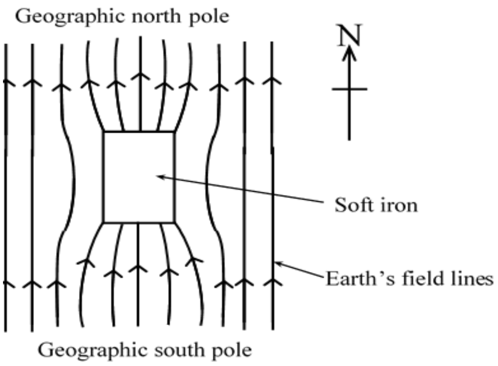
Earth's magnetic field

This is the series of parallel lines running from geographic south to geographic north as shown below.

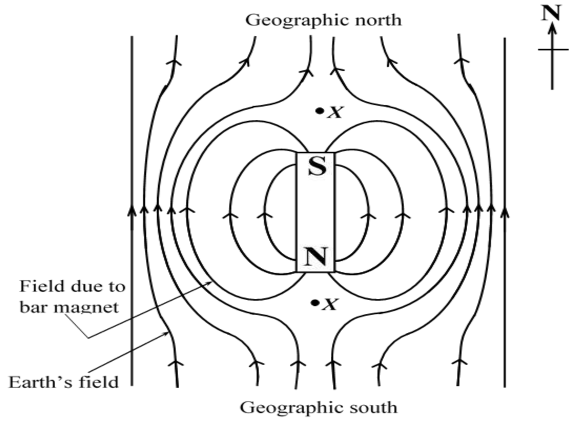


The magnetic field of the earth is uniform, that is the magnetic field lines are parallel to each other.

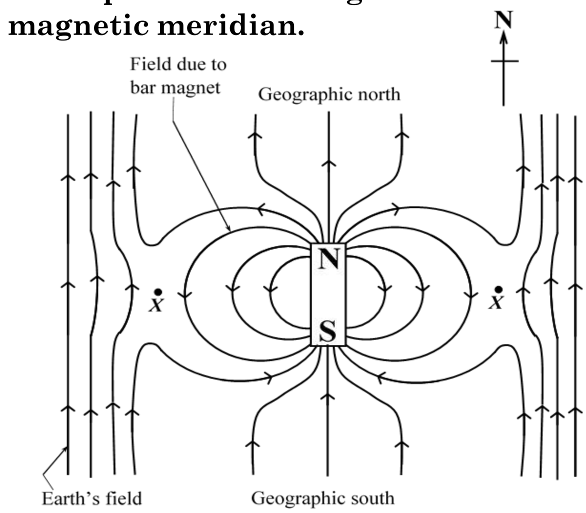
Interaction of earth's field with soft iron.



Interaction of earth's field with a bar magnet when the south pole of the bar magnet is pointing in the geographical north pole and the magnet is in the magnetic meridian.



Interaction of earth's field with a bar magnet when the north pole of the bar magnet is pointing in the geographical north pole and the magnet is in the magnetic meridian.



Note:

At point ***X,*** the magnetic field lines cancel out each other and any magnetic material placed at that point does not experience any magnetic force. ***X*** is therefore called a ***<u>magnetic neutral point</u>***.

Definition:

Magnetic neutral point refers *to a point in the magnetic field where the resultant magnetic force is zero.*

The Molecular (Domain) theory of Ferromagnetism

Every ferromagnetic material has a very strong interaction between the nearby atoms. This creates magnetic fields generated to line-up in the same direction, in different regions, causing the regions to be spontaneously magnetized. These regions are called ***domains***.
The direction of magnetization of the domains vary from domain to domain and in the absence of an external magnetic field, the domains cancel out each other, resulting into a net zero magnetization in the material.

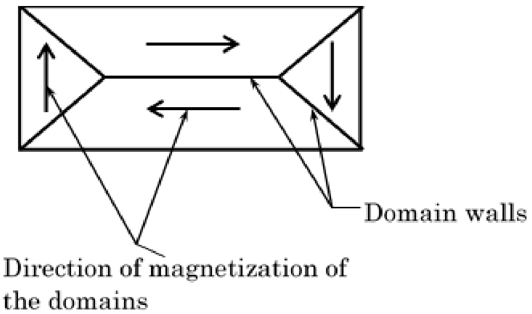
NB

Domain theory helps to explain

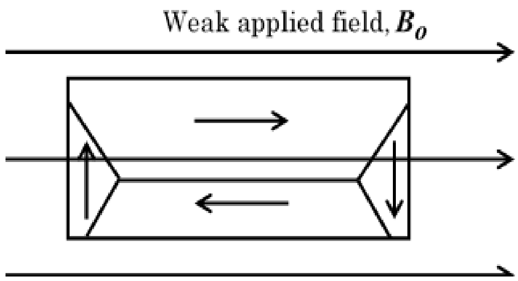
* Magnetization of a material
* Demagnetization of a material
* Hysteresis in magnetic materials.

Magnetization as explained by domain theory.

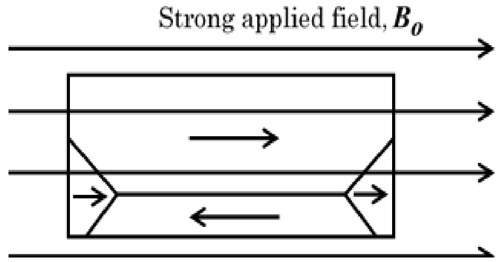
In absence of a an external magnetic field $\boldsymbol{B_o}$, the fields of various spontaneously magnetized domains cancel out each other.



When an external magnetic field, $\boldsymbol{B_o}$ is applied to a ferromagnetic material, the domains whose directions are in the direction of $\boldsymbol{B_o}$, grow at the expense of the others. The domain walls move.



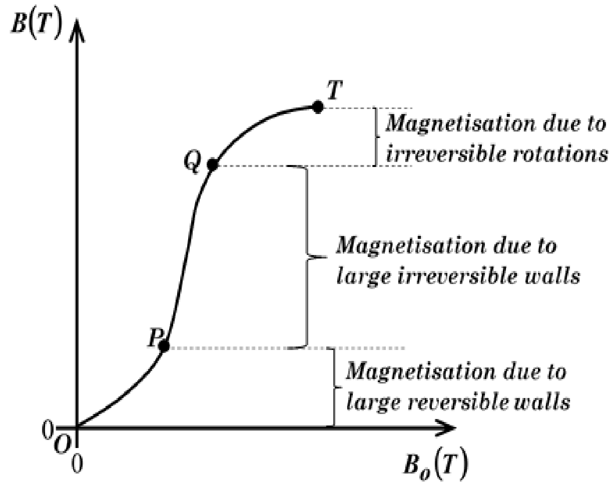
If the external magnetic field is increased more, the domain walls move faster and a point is reached when the domain axis suddenly rotates and lines up with $\boldsymbol{B_o}$.



A father increment of $\boldsymbol{B_o}$ cause more domains to rotate and eventually all the domain axes line up with $\boldsymbol{B_o}$. The material is said to be ***magnetized***.

Magnetization curve.

This shows how the Magnetic induction (Magnetic flux density), $\boldsymbol{B}$ inside the material varies with the applied magnetic field, $\boldsymbol{B_o}$.



Along OP, the magnetization is small and reversible.

Between P and Q, the magnetization increases rapidly as $\boldsymbol{B_o}$ increases and it is irreversible.

Beyond Q, for values of $\boldsymbol{B_o}$, very little increase of $\boldsymbol{B}$ occurs and the material is said to be approaching full magnetisation along QT.

At T, the material is said to be magnetically saturated.

Note

When all the domain axes have been rotated and face in the direction of the external applied magnetic field, then the material is said to be ***magnetically saturated***.

Hysteresis

This refers to *the tendency of the magnetic domains to stay in the current orientation in a material when the direction of the magnetizing field is reversed.*

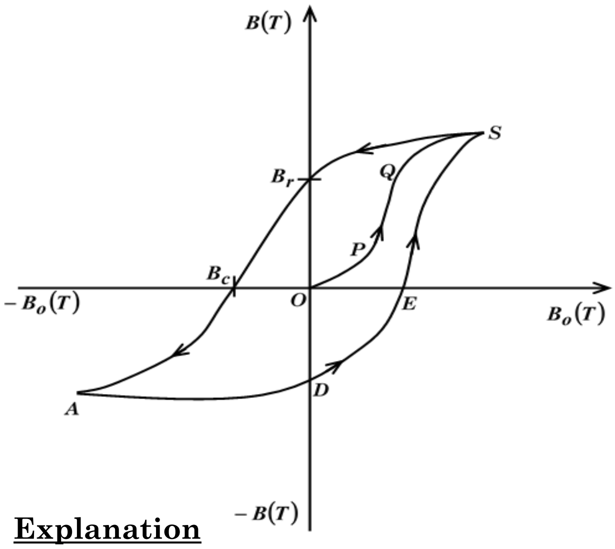
OR,

It *is the lagging of the magnetic induction,* ***B****, in a ferromagnetic material with respect to the cyclic variation of the magnetic field* B_o *applied to the material.*

NB

Hysteresis can be best illustrated using a curve called ***hysteresis curve*** (or ***hysteresis loop***)

Hysteresis curve



Explanation

Along OPQS, the material is being magnetized and so, ***B*** increases as B_o is increased.

When the material has reached saturation (at S) and the magnetizing field B_o is reduced to zero, the material remains strongly magnetized and retains some flux density B_r called ***remanence*** (or ***retentivity***) of the material.

When the magnetizing field B_o is reversed, ***B*** decreases and becomes zero when $B_o = B_c$ (***coercivity*** of the material)

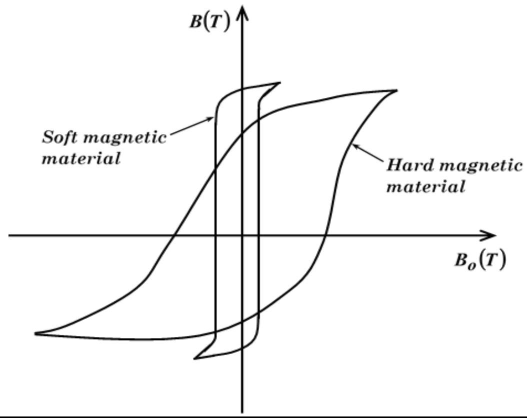
If the reverse magnetizing field is increased more, the material becomes magnetically saturated in reverse direction at ***A***.

Decreasing the field and again reversing to saturation point, S, gives the rest of the loop ***ADES.***

The loop obtained is known as ***hysteresis loop***.

Note

1. Hysteresis curve shows that magnetization, ***B*** of a material lags behind the magnetizing field, B_o when it is taken through a complete magnetization cycle. This effect is known as ***Hysteresis.***
2. The size of the loop is directly proportional to the amount of energy required to take a unit volume of a material through one complete cycle of magnetization. This energy increases the internal energy of the material which is lost as heat to the surroundings and it is known as ***Hysteresis loss.***
3. The hysteresis loop for a hard magnetic material (hard steel) is larger than that of a soft magnetic material (soft iron). This implies that soft iron generates a lower hysteresis loss compared to hard steel and it is for this reason that soft iron is preferred to hard steel in an A.C transformer.



Note:

Reversing B_O to B_C reduces the magnetic flux density of the material to zero but does not permanently demagnetize it.

Therefore, for effective demagnetization of a ferromagnetic material, the material is inserted into a solenoid through which an alternating current is flowing and then either the current is reduced to zero or the material is withdrawn slowly from the solenoid.

In either case, the material is taken through a series of diminishing hysteresis loops.

Magnetic flux, ϕ and Magnetic flux density, B

Magnetic flux density, B.

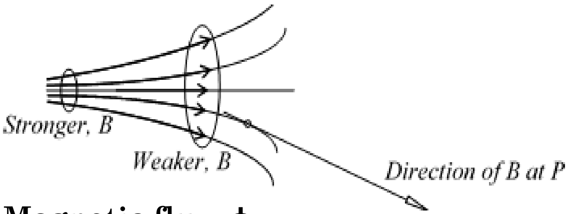
This is the *force acting on a straight conductor of length 1m, carrying a current of 1A and placed perpendicular to a uniform magnetic field.*

The magnetic flux density, ***B*** represents the magnitude and direction of the field. It is at times called ***<u>magnetic induction</u>***.

Magnetic flux density, ***B*** is directly proportional to magnetic field strength, ***H*** i.e $B \propto H$.

The S.I unit of flux density is a ***<u>Tesla, T</u>***.

NB:
The closer the magnetic field lines the stronger the magnetic flux density, B. The direction of B at any point is given by the tangent to the field line at that point.



<u>Magnetic flux, ϕ</u>.

This refers to *the product of the magnetic flux density, B and the area,* ***A,*** *through which the magnetic field lines are passing perpendicularly*. **OR,**

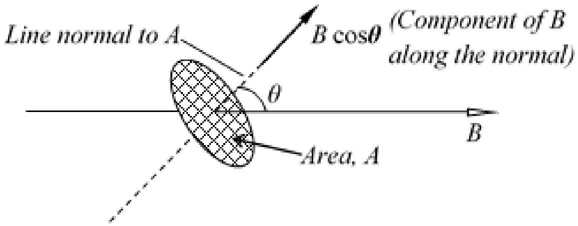
It is *product of the magnetic flux density and the projection of the area normal to the magnetic field.*

i.e., $\phi = B \times A$.

The magnetic flux through a region is the measure of the number of magnetic field lines passing through the region.

<u>Derivation of $\phi = AB\cos\theta$.</u>

Consider a flat circular coil of area, A whose normal makes an angle, θ with a magnetic field of flux density, ***B***.



By definition, flux, $\phi = B \times A$, but $B\cos\theta$ is the component of ***B*** along the normal

$$\Rightarrow \phi = A \times B\cos\theta \quad \therefore \phi = AB\cos\theta$$

The S.I unit of magnetic flux is a ***<u>Webber, Wb</u>***.

NB:
(i) From,

$$\phi = A \times B\cos\theta \quad \Rightarrow \quad B = \frac{\phi}{A \times \cos\theta}.$$

Thus, if A is perpendicular to B then, $\theta = 0^o$ and so,

$$\cos 0 = 1 \Rightarrow B = \frac{\phi}{A} \quad \therefore \quad \phi = A \times B.$$

Therefore, ***magnetic flux density***, *B can be defined as the number of magnetic field lines per unit area (1 m ^{2}) passing through a region perpendicularly.*

(ii) 1 Wb = 1 T m ^{2} or 1 T = 1 Wb m $^{-2}$.

Therefore, a ***Webber*** *refers to the magnetic flux that passes perpendicularly through an area of 1 m ^{2} when the magnetic flux density is 1 T.*

Magnetic fields of a current carrying conductor.

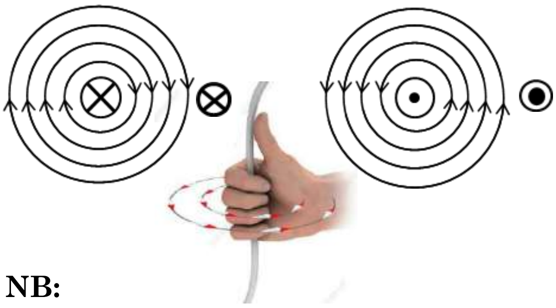
A straight wire carrying an electric current has a magnetic field associated to it and the field lines are a series of concentric circles centred on the wire.

The direction of the field is obtained by the **right hand grip rule**.

The “**Right hand grip rule**” says,

“Grip the wire using the right hand with the thumb pointing in the direction of the current, the fingers then point in the direction of the field.

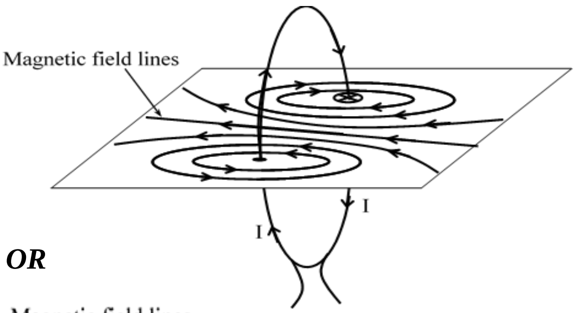
Current into paper | **Current out of the the paper**



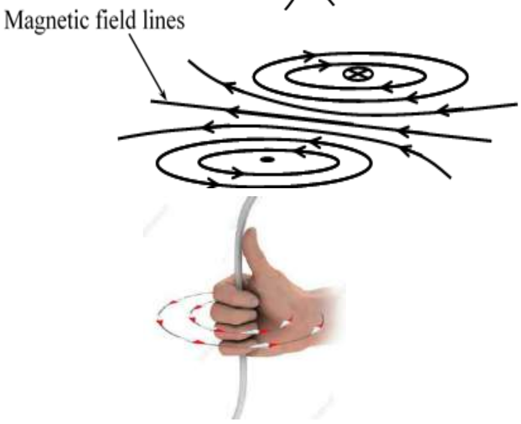
NB:

The field of a wire can be increased by coiling the wire to form a coil or a solenoid.

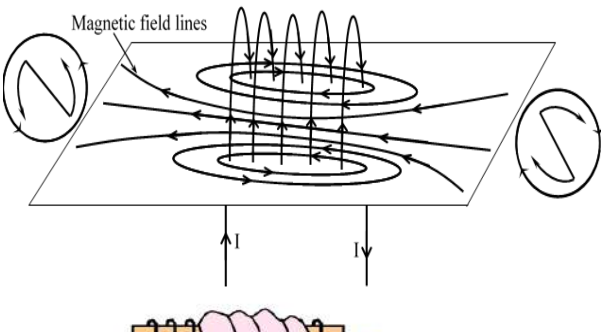
Current through a coil.



OR



Current through a solenoid.



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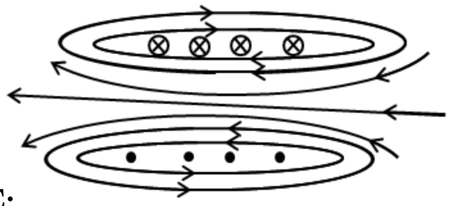


NOTE:

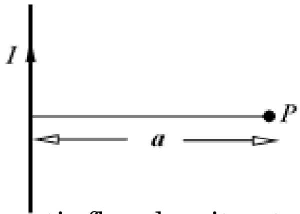
A solenoid produces a magnetic field similar to that of a bar magnet. Inside the solenoid, the magnetic field lines are parallel to the axis of the solenoid and at the ends, the field lines diverge from the axis.

The polarity of the magnet produced can be determined by,

(a) Griping the solenoid with the right hand such that the fingers point in the direction of current then, the **thumb** points to the **North Pole** so that the opposite end is the **South Pole**.

(b) Looking directly at the end of the solenoid, if the current flow is clockwise, the end is a **south pole**. But if the current flow is anti-clockwise then, the pole is a **north pole**.

Magnetic flux density due to an infinitely long straight wire.



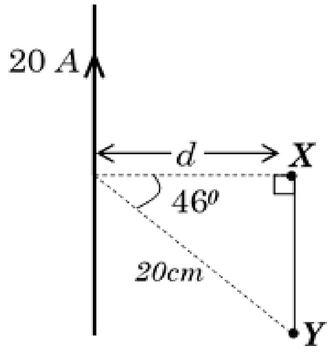
The magnetic flux density at a point P directed into the paper has a magnitude given by the expression

$$B = \frac{\mu_0 I}{2\pi a},$$

where I is the current in Amperes (A) through the wire, **a** is the perpendicular distance in metres (m) of the point P from the wire, μ_o is the constant of proportionality called Permeability of free space ($\mu_0 = 4\pi\times10^{-7}$ H m $^{-1}$.).

NUMERICAL EXAMPLES

1. The figure below shows a wire carrying current of 20A. Determine the magnitude of the magnetic flux density at point ***X***.



Solution

Using

$$\boldsymbol{B} = \frac{\mu_0 \boldsymbol{I}}{2\pi \boldsymbol{a}};\ \mu_0 = 4\pi\times10^{-7}\text{ H m}^{-1},$$

$$\boldsymbol{I} = 20\text{A} \text{ and } \boldsymbol{a} = \boldsymbol{d} = 20\times10^{-2}\cos 46^{o}$$

$$\Rightarrow \boldsymbol{B} = \frac{4\pi\times10^{-7}\times20}{2\pi\times20\times10^{-2}\cos 46^{o}}$$

$$\therefore \boldsymbol{B} = 2.879113\times10^{-5}\boldsymbol{T}$$

2. A child sleeps at an average distance of 30 cm from a household wiring. The mains supply is 240*V* r.m.s. Calculate the maximum possible magnetic flux density in the region of the child when the wire is transmitting 3.6 kW of power.

Solution

Using

$$\boldsymbol{B} = \frac{\mu_0 \boldsymbol{I}}{2\pi \boldsymbol{a}};\ \mu_0 = 4\pi\times10^{-7}\text{ H m}^{-1},$$

$$\boldsymbol{I} = \frac{\boldsymbol{P}}{\boldsymbol{V}} = \frac{3600}{240} = 15\text{A} \text{ and}$$

$$\boldsymbol{a} = 30\times10^{-2}\boldsymbol{m}$$

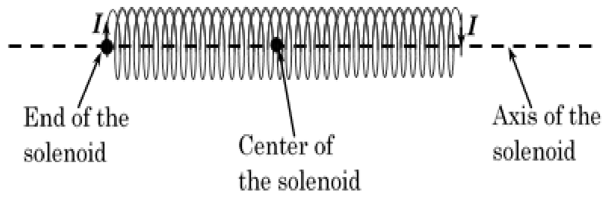
$$\Rightarrow \boldsymbol{B} = \frac{4\pi\times10^{-7}\times15}{2\pi\times30\times10^{-2}} = 1\times10^{-5}\boldsymbol{T},$$

Alternating magnetic field due to alternating current in the wire.

Trial question

Calculate the magnetic flux density ***B*** at a point 120 cm to the west of a long vertical wire carrying current of 1100***mA*** in vacuum.

Magnetic flux density, B on the axis of an infinitely long solenoid.

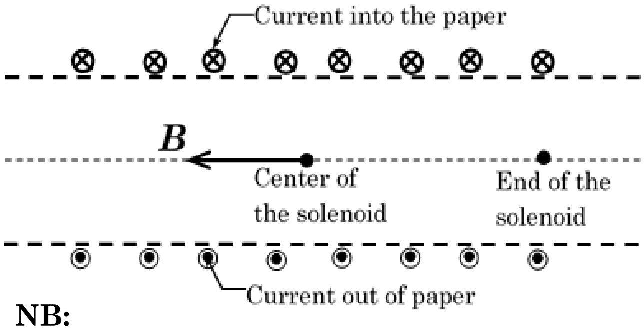


At the centre

The magnetic flux density, ***B*** on the axis of an infinitely long solenoid is directed along the axis and its magnitude near the centre is given by,

$$\boldsymbol{B} = \mu_0 \boldsymbol{n}\,\boldsymbol{I},$$

where ***I*** is the current in amperes (A) through the solenoid, ***n*** is the number of turns per unit length, *l* is the length of the solenoid.



NB:

(i) $\boldsymbol{n} = \frac{\boldsymbol{N}}{\boldsymbol{l}} \Rightarrow \boldsymbol{B} = \frac{\mu_0 \boldsymbol{N}\,\boldsymbol{I}}{\boldsymbol{l}}$, where ***N*** is the number of turns of the solenoid of length, ***l***.

(ii) ***nI*** is called ***ampere turns per metre*** and is equal to the **magnetic field strength**, ***H***. i.e.

$$\boldsymbol{H} = \boldsymbol{nI} \Rightarrow \boldsymbol{B} = \mu_0 \boldsymbol{H}.$$

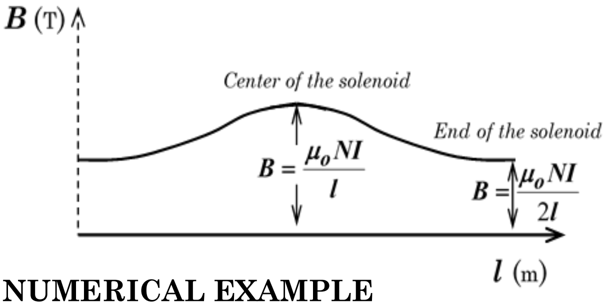
Magnetic field strength *refers to the magnetic force exerted on a unit magnetic pole in a magnetic field.*

The unit of ***H*** is ***Ampere per metre*** (A m^{-1}).

(iii) At either end of a long solenoid, the flux density along the axis is given by,

$$\boldsymbol{B} = \frac{\mu_0 \boldsymbol{n}\,\boldsymbol{I}}{2}.$$

Variation of magnetic flux density, B with length, l of an infinitely long solenoid



NUMERICAL EXAMPLE

1. A 2000 turn solenoid of length 50 cm and resistance 25Ω is connected to a 15 V supply. Determine the magnetic flux density at the mid-point on the axis of the solenoid.

Solution

Using $B = \frac{\mu_o NI}{l}$; $I = \frac{V}{R}$,

$$\Rightarrow I = \frac{15}{25} = 0.6A$$

$\mu_0 = 4\pi \times 10^{-7}\ \text{Hm}^{-1}$,

$$\Rightarrow B = \frac{4\pi \times 10^{-7} \times 2000 \times 0.6}{50 \times 10^{-2}}$$

$$\therefore \quad B = 3.0159289 \times 10^{-3} T$$

2. A solenoid of length 35 cm with an iron core wound with 100 turns of a wire carries a current of 2***A***. If the flux density of 4.5***T*** is produced at the centre of the solenoid, calculate the relative permeability of the core and the magnetic field strength produced. (*Assume* $\mu_0 = 4\pi \times 10^{-7}\ \text{Hm}^{-1}$)

Solution

Using $B = \mu_m n I$, where $\mu_m = \mu_r \mu_o$

and μ_r is relative permeability of the core.

$$\Rightarrow 4.5 = \mu_r \left(\frac{4\pi \times 10^{-7} \times 100 \times 2}{35 \times 10^{-2}} \right)$$

$$\Rightarrow \mu_r = \frac{4.5 \times 35 \times 10^{-2}}{4\pi \times 10^{-7} \times 100 \times 2}$$

$$\therefore \mu_r = 6.266725 \times 10^3.$$

3. A 2000 turn solenoid of length 40 cm and resistance 16Ω is connected to a 20*V* supply. Determine the flux density on the axis of the solenoid at the

(a) centre of the solenoid.
(b) end of the solenoid.

Solution

(a) Using $B = \frac{\mu_o NI}{l}$; $I = \frac{V}{R}$,

$$\Rightarrow I = \frac{20}{16} = 1.25A$$

$\mu_0 = 4\pi \times 10^{-7}\ \text{Hm}^{-1}$,

$$\Rightarrow B = \frac{4\pi \times 10^{-7} \times 2000 \times 1.25}{40 \times 10^{-2}}$$

$$\therefore \quad B = 7.853982 \times 10^{-3} T$$

(b) Using $B = \frac{\mu_o NI}{2l}$; $I = \frac{V}{R}$,

$$\Rightarrow I = \frac{20}{16} = 1.25A$$

$\mu_0 = 4\pi \times 10^{-7}\ \text{Hm}^{-1}$,

$$\Rightarrow B = \frac{4\pi \times 10^{-7} \times 2000 \times 1.25}{2 \times 40 \times 10^{-2}}$$

$$\therefore \quad B = 3.926991 \times 10^{-3} T$$

4. A copper wire of length 15.7 m is wound into a solenoid of radius 5.0 cm and length 25 cm. A current of 1.5 A is passed through the coil. Calculate the magnetic flux density at the centre of the solenoid.

Solution

Using $B = \frac{\mu_o NI}{l}$, $\Rightarrow I = \frac{20}{16} = 1.25A$

$\mu_0 = 4\pi \times 10^{-7}\,\text{Hm}^{-1}$,

$N = \frac{\text{Length, } L \text{ of the wire}}{\text{Length, } C \text{ of one turn}}$; where

l=15.7m and $C = 2\pi r = 2 \times \frac{22}{7} \times 5 \times 10^{-2}$

$\therefore C = 3.14285714 \times 10^{-1} m$

$\Rightarrow N = \frac{15.7}{3.14285714 \times 10^{-1}} \approx 50 turns$

$\Rightarrow B = \frac{4\pi \times 10^{-7} \times 50 \times 1.5}{25 \times 10^{-2}}$

$\therefore \quad B = 3.769911 \times 10^{-4} T$.

5. A current of 1.0A flows in a long solenoid of 1000 turns per meter. If the solenoid has a mean diameter of 80cm, find the maximum magnetic flux linkage on one-meter length of the solenoid.

Solution

Maximum flux linkage per metre is given by,

$\phi_{max} = nA \times B_{max} \cos\theta$

But the flux density due to the solenoid is maximum at the centre of the solenoid.

At the centre of the solenoid, the flux is along the axis of the solenoid(i.e. parallel to the normal to the plane of each loop) and so,

$\theta = 0^o$ thus, $\cos\theta = \cos 0^o = 1$

$B_{max} = \mu_o nI$

$B_{max} = 4\pi \times 10^{-7} \times 1000 \times 1.0$

$\therefore \quad B_{max} = 1.256637 \times 10^{-3} T$

$A = \frac{\pi d^2}{4} = \frac{\pi \times (80 \times 10^{-2})^2}{4}$

$\therefore \quad A = 0.502655 m^2$

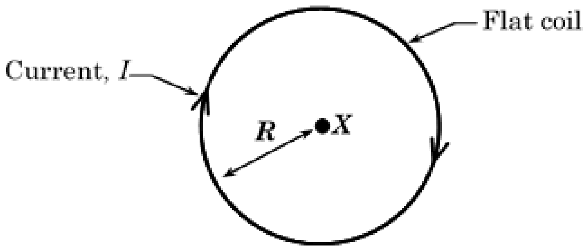
$\phi_{max} = 1000 \times 0.5027 \times 1.2566 \times 10^{-3}$

$\therefore \phi_{max} = 6.3169 \times 10^{-1} Wb$

Trial questions

1. A fine wire of length 157.0m is wound into a solenoid of diameter 5.0cm and length 25cm. If a current of 2.0A passes through the coil, find the magnetic flux density at the end of the solenoid. (***Ans*** $5.0264 \times 10^{-3} T$)
2. A solenoid of 2000 turns, 40cm long and resistance of 16Ω is connected to a 20V supply. Calculate the magnetic flux density at;
   (i) The midpoint of the axis of solenoid.
   (***Ans*** $7.853982 \times 10^{-3} T$)
   (ii) At the end of the solenoid
   (***Ans*** $3.926991 \times 10^{-3} T$)
3. A solenoid of 2000 turns, 75cm long and carrying a current of 2.5A. Calculate the magnetic flux density at;
   (i) Center of the solenoid
   (***Ans*** $8.377580 \times 10^{-3} T$)
   (ii) End solenoid
   (***Ans*** $4.188790 \times 10^{-3} T$)

The magnetic flux density at centre of a plane circular coil.



The magnetic flux density, B at the centre point X of a plane (flat) circular coil, directed into the paper according to the right hand grip rule is given by,

$$B = \frac{\mu_0 N I}{2R},$$

where, I is the current through the coil, R is the radius of the coil and μ_o is the permeability of free space and N is the number of turns of the coil.

NUMERICAL EXAMPLE

1. A copper wire of length 157 m is wound into coil of radius 10 cm. A current of 1.5 A is passed through the coil. Calculate the magnetic flux density at the centre of the coil.

Solution

Using $\boldsymbol{B} = \dfrac{\mu_0 \boldsymbol{N} \boldsymbol{I}}{2\boldsymbol{R}}$; $\boldsymbol{N} = \dfrac{\boldsymbol{L}}{\pi \boldsymbol{d}}$

$$\Rightarrow \boldsymbol{N} = \frac{157}{\pi \times 20 \times 10^{-2}} \approx 250 \textbf{\textit{turns}}$$

$$\boldsymbol{B} = \frac{4\pi \times 10^{-7} \times 250 \times 1.5}{2 \times 10 \times 10^{-2}}$$

$$\therefore \quad \boldsymbol{B} = 2.356194 \times 10^{-3} \boldsymbol{T}.$$

2. Calculate the current that must be passed through a flat circular coil of 10 turns and radius 5.0 cm to produce a flax density of $2.0 \times 10^{-4} \boldsymbol{T}$ at its centre.

Solution

Using $\boldsymbol{B} = \dfrac{\mu_0 \boldsymbol{N} \boldsymbol{I}}{2\boldsymbol{R}}$;

$$\Rightarrow 2.0 \times 10^{-4} = \frac{4\pi \times 10^{-7} \times 10 \times \boldsymbol{I}}{2 \times 5.0 \times 10^{-2}}$$

$$\therefore \quad \boldsymbol{I} = 1.591549 \boldsymbol{A}.$$

3. An electron revolves in a circular orbit of radius $2.0 \times 10^{-10} \boldsymbol{m}$ at a frequency of 6.8×10^{15} revolutions per second. Calculate the magnetic flux density at the centre of the orbit.

Using $\boldsymbol{B} = \dfrac{\mu_0 \boldsymbol{N} \boldsymbol{I}}{2\boldsymbol{R}}$; $\boldsymbol{N} = 1$

A revolving electron constitutes current, ***I*** flowing in opposite direction.

$$\boldsymbol{I} = \text{charge flowing per second}$$

$$\boldsymbol{I} = \boldsymbol{ef} = 1.6 \times 10^{-19} \times 6.8 \times 10^{15}$$

$$\boldsymbol{I} = 1.088 \times 10^{-3} \boldsymbol{A}$$

$$\Rightarrow \boldsymbol{B} = \frac{4\pi \times 10^{-7} \times 1 \times 1.088 \times 10^{-3}}{2 \times 2.0 \times 10^{-10}}$$

$$\therefore \quad \boldsymbol{B} = 3.4180528 \boldsymbol{T}.$$

Trial questions

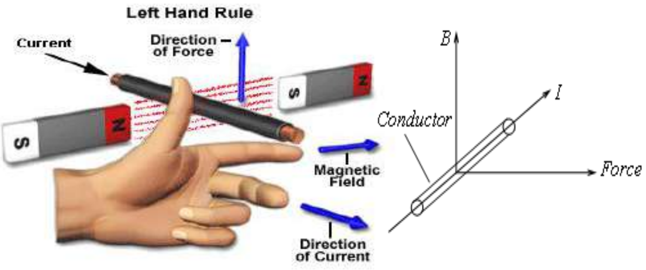
1. A wire of length 7.85 m is wound into a circular coil of radius 0.05 m. If a current of 2***A*** passes through the coil, find the magnetic flux density at the centre of the coil.
   (***Ans*** $1.256637 \times 10^{-4} \boldsymbol{T}$)
2. A circular coil of 10 turns and radius 5.0 cm carries a current of 1.0***A*****.** Find the magnetic flux density at its centre.
   (***Ans*** $6.28 \times 10^{-4} \boldsymbol{T}$)
3. A coil of conducting wire has 600 turns and diameter of 10.0 cm. If a current of 1.2***A*** is passed through it, determine the magnetic flux density at its centre.
   (***Ans*** $9.0 \times 10^{-3} \boldsymbol{T}$)
4. Calculate the magnetic flux density at the centre of a flat circular coil of 600 turns having a radius of 20 cm if a current of 0.4*A* is passed through it.
   (***Ans*** $7.5398 \times 10^{-4} \boldsymbol{T}$)
5. A force of $3.037 \times 10^{-13} \boldsymbol{N}$ acts on an electron making it to revolve a circular orbit at a speed of $2.0 \times 10^{7} \boldsymbol{ms}^{-1}$ at a frequency of 1.2×10^{18} revolutions per minute. Determine the magnetic flux density generated at the centre of the orbit.
   (***Ans*** $33.514003 \boldsymbol{T}$)

<u>Force on a current carrying conductor in a magnetic field</u>

When a wire carrying current, ***I*** is placed in a magnetic field, it experiences a mechanical force perpendicular to the plane of the of the current and the field, ***B***.

The direction of the force acting on the wire can be predicted by using the **"*Flemings Left Hand rule*"**, also known as the **"*The Motor rule*"**.

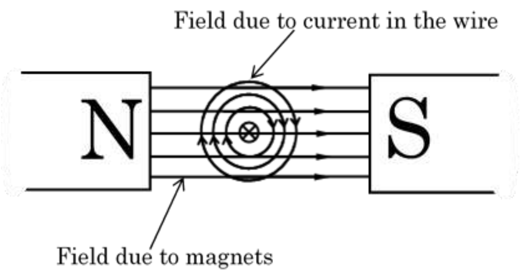
<u>The rule states that</u> "*If the thuMb, First finger and the seCond of the left hand are stretched out at right-angles to one another, the First figure points in the direction of the magnetic Field, the seCond finger points in the direction of the Current and the thuMb shall point in the direction of Motion (the Force)*".



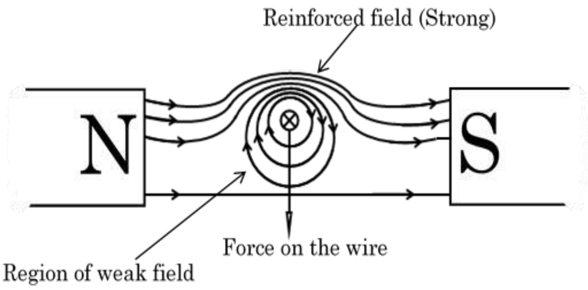
Origin of the force on the conductor.

A current carrying conductor has a magnetic field around itself.
When such a conductor is placed in another magnetic field, the two magnetic fields interact.
Since the two fields above the wire are in the same direction, they reinforce and the resultant field becomes stronger.
Since the two fields below the wire are in opposite directions, they tend to cancel out each other and the resultant field becomes weaker.
If we suppose the field lines to be stretched elastic materials (**catapult field)**, then the field lines above the wire will try to straighten out and in so doing, they exert a downward force on the wire.

(a) Before interaction



(b) After interaction



NB:
If a current carrying conductor is placed in the same direction as a uniform magnetic field, the flux density on both sides is the same and therefore, no resultant force acts hence no motion.

Factors affecting magnetic force.

(i) Magnitude of the current, I through the conductor. $F \propto I$.
(ii) Length, l of the conductor in the magnetic field. $F \propto l$.
(iii) Magnitude of the magnetic flux density, B. $F \propto B$.
(iv) The orientation of the conductor in the magnetic field of the angle, θ between the conductor and the magnetic field, B. $F \propto \sin\theta$.
Combining all the factors above, we have,

$$F \propto BIl\sin\theta \Rightarrow F = kBIl\sin\theta \ldots(*)$$

where k is the constant of proportionality.
But the unit of B is a **Tesla**.

A **<u>Tesla</u>** refers to *the magnetic flux density of a uniform field when the force on a conductor, **1 m** long placed perpendicular to the field and carrying a current of **1 A** is **1 N**.*

From the definition,
$F = 1N, B = 1T, I = 1A, l = 1m$
and $\theta = 90^o$.

Substituting the values into (*), we have;
$1 = k \times 1 \times 1 \times 1 \times \sin 90^o \Rightarrow k = 1$.
$\therefore \quad F = BIl\sin\theta$.

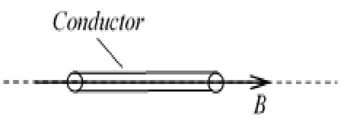
NB:

(i) When the conductor is placed perpendicular to the field, B it experiences maximum force given by $F_{max} = BIl$, since $\theta = 90^o$.

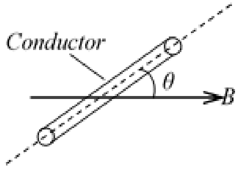
(ii) When the conductor is parallel to the field, $\theta = 0^o$ $F = 0N$, therefore he conductor experiences non force.

Illustrations.

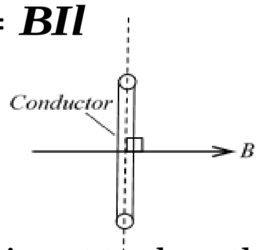
(a) $F = 0N$



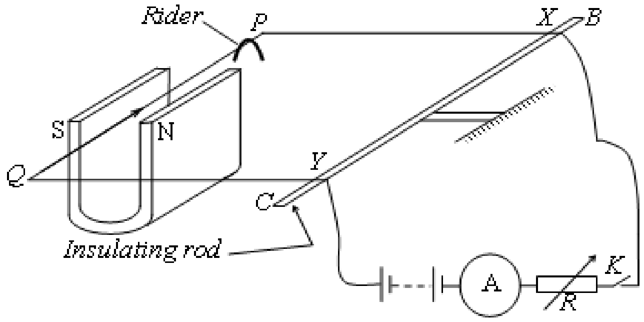
(b) $F = BIl\sin\theta$



(c) $F = BIl$



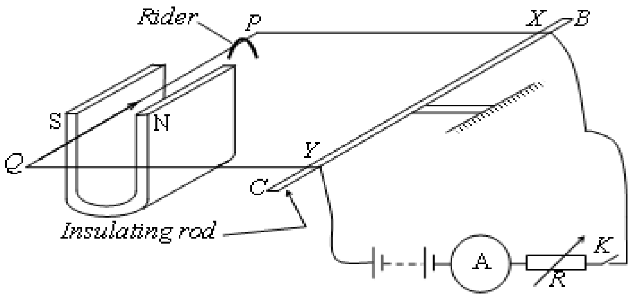
An experiment to show that $F \propto l$



XPQY is a conducting frame and ***BC*** is an insulating rod that acts like a pivot.

The frame is adjusted so that its plane is horizontal.
A variable resistor, ***R*** is adjusted so that the current in the frame is suitably small.
When switch, ***K*** is closed, the arm ***PQ*** rises.
Riders of known mass are placed on arm ***PQ*** to restore horizontal balance.
When the current is increased, more riders have to be added to the arm ***PQ*** to restore horizontal balance.
This means that the magnetic force on ***PQ*** is proportional to the current, I through the wire ***PQ*** i.e $F \propto I$

An experiment to show that $F \propto I$



XPQY is a conducting frame and ***BC*** is an insulating rod that acts like a pivot.

The frame is adjusted so that its plane is horizontal.
A variable resistor, ***R*** is adjusted so that the current in the frame is suitably small.
With only one **U**-shaped magnet, switch, ***K*** is closed and the arm ***PQ*** rises.
With the value of ***R*** fixed, the number of magnets of identical size and strength are placed along ***PQ***.
The length of the wire in the field is therefore increased. More riders have to be added to restore horizontal balance.
This means that the magnetic force on the conductor is proportional to the length of the conductor in the field. i.e. $F \propto l$.

NB:

(i) In order to show that $F \propto B$, the bar magnets are replaced by the by an electromagnet made by passing current through a coil of many turns. As the current in the coil is increased, the magnetic flux density also increases and hence, more riders are added to restore equilibrium. This means that, $F \propto B$.

(ii) For a fixed value of current in the coil, the orientation of ***PQ*** with the magnetic field at the centre of the coil is varied. The number of riders required to restore balance is greatest when ***PQ*** is perpendicular to the field.

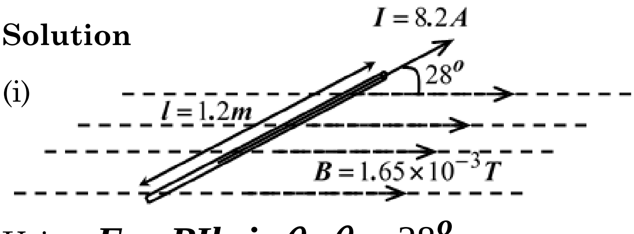
NUMERICAL EXAMPLE

1. A conductor carrying a current of 8.2 A and of length 1.2 m is placed in a magnetic field of flux density $1.65\times10^{-3}T$. Determine the force on the conductor when it is

   (i) at an angle of 28^{o} to the field.
   (ii) perpendicular to the field.
   (iii) parallel to the field.

Solution

(i)

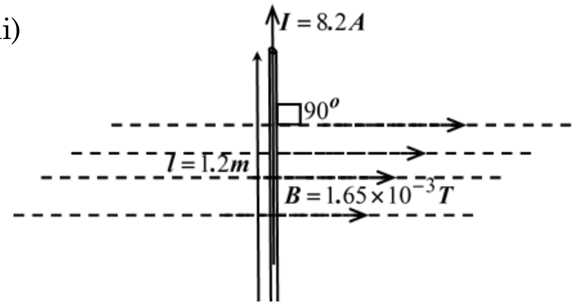


Using $F = BIl\sin\theta$; $\theta = 28^{o}$

$$F = 1.65\times10^{-3}\times8.2\times1.2\times\sin 28^o$$

$$\therefore \quad F = 7.622343\times10^{-3}N$$

(ii)

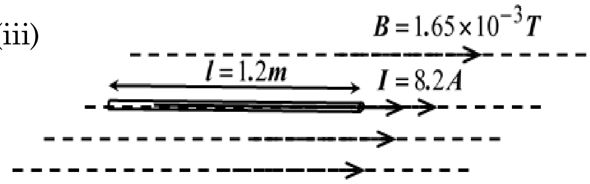


Using $F = BIl\sin\theta$; $\theta = 90^o$

$$F = 1.65\times10^{-3}\times8.2\times1.2\times\sin 90^o$$

$$\therefore \quad F = 1.6236\times10^{-2}N$$

(iii)



Using $F = BIl\sin\theta$; $\theta = 0^o$

$$F = 1.65\times10^{-3}\times8.2\times1.2\times\sin 0^o$$

$$\therefore \quad F = 0N$$

2. A horizontal wire carrying a current of 5.0A lies in a vertical magnetic field of flux density0.05T. Calculate the force per meter on the wire.

Using $F = BIl\sin\theta \Rightarrow \frac{F}{l} = BI\sin\theta$;

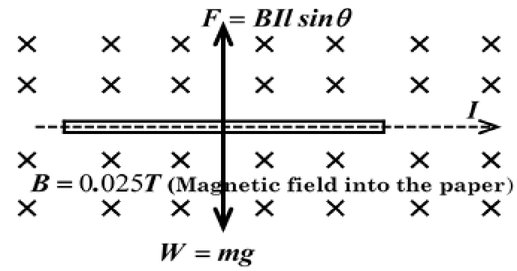
But $\theta = 90^o$

$$\Rightarrow \frac{F}{l} = 0.05\times5\times\sin 90^o$$

$$\therefore \frac{F}{l} = 0.25Nm^{-1}.$$

3. A horizontal straight conducting wire of mass 50mg and length 50 cm is placed in a uniform magnetic field of 0.025T. If the field is perpendicular to the conductor, calculate the current flowing through the conductor if the conductor is in equilibrium under the action of its weight and the magnetic force on it.

Solution



At equilibrium, $mg = BIl\sin\theta$;

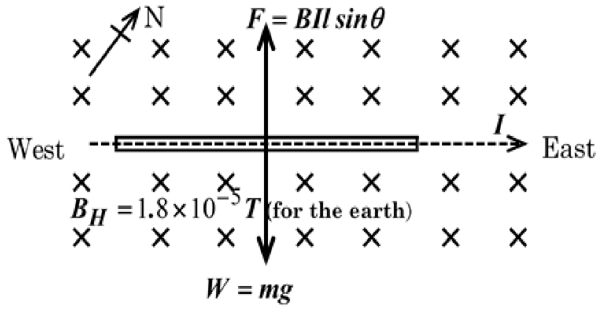
$\theta = 90^o$

$$\Rightarrow 0.05\times9.81 = 0.025\times I\times0.5\times\sin 90^o$$

$$\therefore \quad I = 39.24A$$

4. A metal wire 10 m long lies east-west on a wooden table. What p.d, would have to be applied to the ends of the wire, in order to make the wire rise from the surface. (*Use; density of metal* $=1.0\times10^4 kgm^{-3}$, resistivity of the metal $=2.0\times10^{-8}\Omega m$, horizontal component of earth's magnetic field = $1.8\times10^{-5}T$ and $g = 9.81ms^{-2}$)

Solution



At equilibrium, $mg = BIl\sin\theta$;

$\theta = 90^o$ and $m = \rho' Al$, $I = \frac{V}{R}$; where

$$R = \frac{\rho l}{A} \Rightarrow \rho' Alg = \frac{B_H lVA\sin 90^o}{\rho l}$$

$\Rightarrow \rho' lg = \frac{B_H V}{\rho}$ since $I = \frac{VA}{\rho l}$. But,

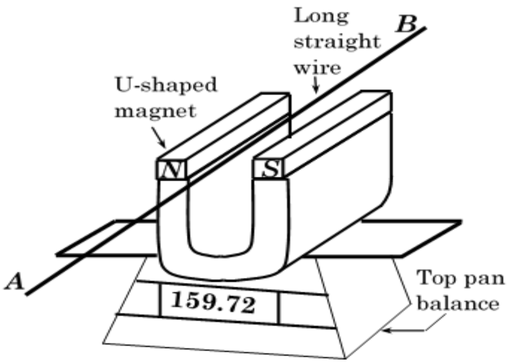
$\rho' = 1.0\times10^4 kgm^{-3}$,

$\rho = 2.0\times10^{-8}\Omega m$, $B_H = 1.8\times10^{-5}T$.

$$\Rightarrow 1.0\times10^4\times10\times9.81=\frac{1.8\times10^{-5}V}{2.0\times10^{-8}}$$

$\therefore\quad V=1090\,V$.

5. A U-shaped magnet sits on a top pan balance as shown below, with a wire ***AB*** fixed horizontally between the poles of the magnet such that a length of 4.20 cm lies in the magnetic field.



Without current in the wire, the balance reading is 159.72g and the balance reads 159.46g when a current of 2.0*A* flows through the wire.

(a) (i) Account for the difference in the balance reading.
(ii) State the direction of flow of current in the wire.

(b) Determine the
(i) value of the magnetic field strength at the wire.
(ii) reading of the balance if the direction of current is reversed.

(c) Suggest how a periodic force could be produced on the pan of the balance.

Solution

(a) (i) With no current flowing in the wire, the only force on the balance is due to the mass of the magnet.
When a current made to flow through the wire, a magnetic force acts on the wire upwards according to Fleming's left hand rule.
This creates a slight on a magnet causing the reading of the balance to change.

(ii) According to Flemings's left hand rule, current is flowing from end ***B*** to end ***A***

(b) (i) At equilibrium;

Magnetic force On the wire = Apparent loss in weight of the balance.

$$\Rightarrow BIl=\left(m-m^{/}\right)g$$

$$B\times2\times0.042=\left(\frac{159.72-159.46}{1000}\right)\times9.81$$

$$\therefore B=3.095238\times10^{-3}T.$$

(ii) Reversing the direction of current also reverses the direction of the magnetic force on the wire. The wire is pulled down wards

New reading of the balance is

$$m^{//}=159.72+(159.72-159.46)$$

$$m^{//}=159.98g$$

(c) The periodic force can be produced by passing an alternating current through the wire.

<u>Trial questions</u>

1. A conductor of length 10 cm carrying a current of 5 A is placed in magnetic field of flux density 0.2***T.*** calculate the force on the conductor when placed;
   (i) at right angle to the field (***Ans*** 0.1 ***N***)
   (ii) at 30^o to the field (***Ans*** 0.087 ***N***)
   (iii) parallel to the field (***Ans*** 0 ***N***)

2. A straight horizontal wire 5 cm long weighing $1.2gm^{-1}$ is placed normal to a uniform horizontal magnetic field of flux density 0.6***T***. if the resistance of the wire is $3.8\,\Omega m^{-1}$. Calculate the p.d that has to be applied between the ends of the wire to make it just self-supporting. (***Ans*** $3.7278\times10^{-3}V$)

3. A straight horizontal rod of mass 140 g and length 0.6 m is placed in a uniform horizontal magnetic field 0.16***T*** perpendicular to it. Calculate the current through the rod if the force acting on it just balances its weight. (***Ans*** 14.30625 ***A***)

4. A copper cable of diameter 25 cm carries a current of 2000 A through the earth's magnetic field of $3.0\times10^{-5}\boldsymbol{T}$. If the earth's field makes an angle of 42^{o} with the normal to the conductor and the length of the cable is 50 m, determine the force acting on the cable. (***Ans.*** 126 ***N***)

5. A 10 cm long portion of a straight wire carrying a current of 10 A is placed in a magnetic field of 0.1 ***T*** making an angle of 53° with the wire. What magnetic force does the wire experience? (***Ans.*** $7.986355\times10^{-2}\boldsymbol{N}$)

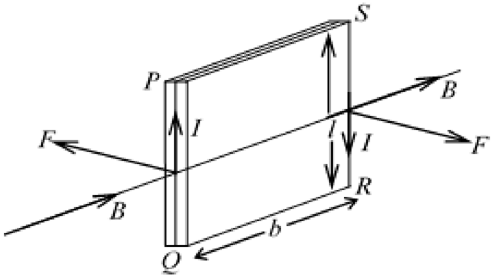
6. At a certain point on the earth's surface, the horizontal component of the earth's magnetic field is $1.8\times10^{-5}\boldsymbol{T}$. A straight piece of conducting wire 2.0 m long, of mass 1.5g, lies on a horizontal wooden bench in an east-west direction. When a very large current flows momentarily in the wire it is just sufficient to cause the wire to lift up off the surface of the bench.

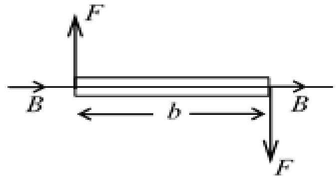
(a) State the direction of the current in the wire. (***Ans. The current is towards the east according to Fleming's left hand rule since the earth's magnetic field is horizontally into the paper.***)

(b) Calculate the value of this current. (***Ans.*** 408.75 ***A***)

Torque on a rectangular coil in a magnetic field.

Consider a rectangular coil made of copper situated with its plane parallel to a uniform magnetic field of flux density, ***B*** and carrying a current, ***I*** through it as shown below.





If the coil has ***N*** turns and sides ***PQ*** and ***RS*** are of length, ***l*** then, the force on either sides of ***PQ*** and ***RS*** is given by, $F = BINl$.
The two forces on side ***PQ*** and ***RS*** together form a couple whose turning effect is called torque, τ. Torque, τ is given by,

$$\tau = F\times b \Rightarrow \tau = BINlb = BIN(lb),$$

But, $lb = A$, area of the coil.

$$\therefore \quad \tau = BINA.$$

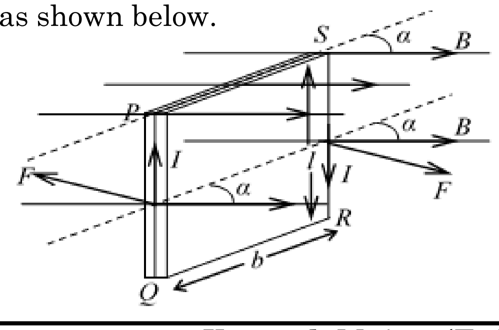
NB:

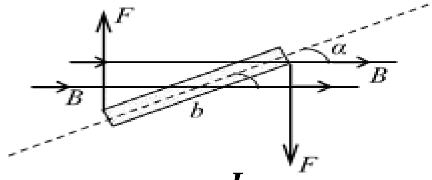
* The unit of torque is **newton metre** (Nm). Another unit is **Webber Ampere** (Wb A).

* There are no forces on PS and QR because they are parallel to the field.

Torque on a rectangular coil whose plane is inclined at an angle, α to the magnetic field.

Consider a rectangular coil with its plane inclined at an angle, ***a*** to the magnetic field of flux density, ***B***, when it is carrying a current, ***I*** as shown below.





The component, B^I of the flux density, B, along the plane of the coil is given by,

$$B^I = B\cos\alpha$$

Therefore, the force, F on side PQ which is equal to the force on side RS is given by,

$$F = B^I INl .$$

Torque, τ is given by,

$$\tau = F \times b \Rightarrow \tau = B^I INlb$$

$$\Rightarrow \tau = B\cos\alpha\left(INlb\right),$$

But, $lb = A$, area of the coil.

$$\therefore \ \tau = BINA\cos\alpha .$$

NB:

(i) When the plane of the coil is parallel to the flux density, B then,

$$\alpha = 0^o \ \Rightarrow \cos\alpha = \cos 0^o = 1.$$

$$\therefore \ \tau = BINA .$$

This is the expression for the **maximum torque** on the coil.

(ii) When the plane of the coil is perpendicular to the flux density, B then,

$$\alpha = 90^o \ \Rightarrow \cos\alpha = \cos 90^o = 0$$

$$\therefore \ \tau = 0\,WbA .$$

(iii) If the angle between the field of flux density, B and the normal to the plane of the coil is θ then,

$$\alpha = \left(90^o - \theta\right)$$

$$\Rightarrow \cos\alpha = \cos\left(90^o - \theta\right) = \sin\theta$$

and therefore, $\tau = BINA\sin\theta$.

Hence, torque on a coil depends on; -

- The magnitude of magnetic flux density, B.
- Current, I through it.
- Its area, A and
- Its orientation in the magnetic field.

Electromagnetic moment (m).

This refers to the torque exerted on a conductor (coil) when it is placed with its plane parallel to a uniform magnetic field of flux density **one** tesla (1T).

$\therefore \ \tau = m$, when $\theta = 0^o$ and $B = 1T$. But $\tau = BINA\cos\alpha \quad \therefore \ m = NIA$.

NB

Electromagnetic moment is sometimes called ***magnetic moment of a current carrying coil*** or ***magnetic dipole moment*** of the coil

NUMERICAL EXAMPLES

1. A circular coil of 10 turns each of radius 10 cm is suspended with its plane along a uniform magnetic field of flux density 0.1T. Find the initial torque on the coil when a current of 1.0A is passed through it.

Solution.

Using $Torque, \tau = BINA\cos\alpha$; where

$$\alpha = 0^o \Rightarrow \cos\alpha = \cos 0 = 1$$

$$\tau = 0.1\times1\times10\times\left(\pi\times(10\times10^{-2})^2\right)$$

$$\therefore Torque, \tau = 3.141593\times10^{-2}\ Nm$$

2. A vertical coil of area 50 cm ^{2} has 80 turns. It is placed in a horizontal magnetic field of magnetic flux density $2.5\times10^{-2}T$ and carries a current of 30 mA. Determine the moment of the couple acting on the coil when the
   (i) plane of the coil makes an angle of 32^o with the field.
   (ii) plane of the coil is parallel to the field.
   (iii) normal to the plane of the coil is inclined at an angle of 72^o to the field.

Solution

(i) Using $Torque, \tau = BINA\cos\alpha$;

where $\alpha = 32^o$

$$\tau = 0.025\times0.03\times80\times50\times10^{-4}\times\cos 32^o$$

$$\therefore Torque, \tau = 2.544144\times10^{-4}\ Nm$$

(ii) Using $\boldsymbol{Torque, \tau = BINA\cos\alpha}$;

where $\alpha = 0^o \Rightarrow \cos\alpha = \cos 0 = 1$

$\tau = 0.025 \times 0.03 \times 80 \times 50 \times 10^{-4} \times 1$

$\therefore \boldsymbol{Torque, \tau} = 3.0 \times 10^{-4}\ \boldsymbol{Nm}$

(iii) Using $\boldsymbol{Torque, \tau = BINA\cos\alpha}$;

where $\alpha = (90^o - 72^o) = 18^o$

$\tau = 0.025 \times 0.03 \times 80 \times 50 \times 10^{-4} \times \cos 18^o$

$\therefore \boldsymbol{Torque, \tau} = 2.853170 \times 10^{-4}\ \boldsymbol{Nm}$

3. A vertical square coil of sides 15 cm has 200 turns and carries a current of 2A. If the coil is placed in a horizontal magnetic field of flux density 0.3***T*** with its plane making an angle of 30^o to the field, find the initial torque on the coil.

Solution

Using $\boldsymbol{Torque, \tau = BINA\cos\alpha}$; where

$\alpha = 0^o \Rightarrow \cos\alpha = \cos 0 = 1$

$\tau = 0.025 \times 0.03 \times 80 \times 50 \times 10^{-4} \times 1$

$\therefore \boldsymbol{Torque, \tau} = 3.0 \times 10^{-4}\ \boldsymbol{Nm}$

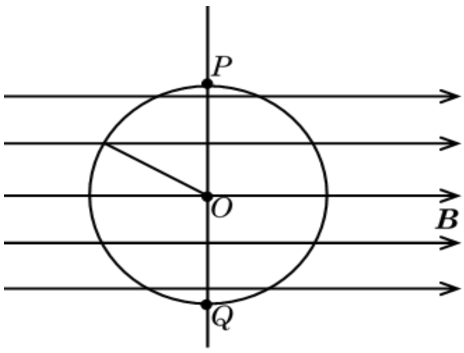
(iii) Using $\boldsymbol{Torque, \tau = BINA\cos\alpha}$;

where $\boldsymbol{A} = \frac{15 \times 15}{10000} = 0.0225\boldsymbol{m}^2$

$\tau = 0.3 \times 2 \times 200 \times 0.0225 \times \cos 30^o$

$\therefore \boldsymbol{Torque, \tau} = 2.338269\ \boldsymbol{Nm}$

4. A circular loop of wire of radius ***r*** is placed in a uniform field of flux density, ***B***, with its axis to the field as shown below.



Explain what happens to the loop when the current starts to flow in it in a

clockwise direction if the loop is pivoted about the axis ***POQ***.

Solution

By Fleming's left hand rule, the left hand side of the loop experiences an outward magnetic force and the right hand side experiences an equal inward force. The two forces constitute a couple which creates a rotational motion of the loop about a vertical axis ***PQ*** in an anticlockwise direction.

5. A small circular coil of 10 turns and mean radius 2.5 cm is mounted at the centre of a long solenoid of 750 turns per metre with its axis at right angles to the axis of the solenoid. If the current in the solenoid is 2.0A, calculate the initial torque on the circular coil when a current of 1.0***A*** is passed through it.

Solution

Using $\boldsymbol{Torque, \tau = BINA\cos\alpha}$; where

$\alpha = 0^o \Rightarrow \cos\alpha = \cos 0 = 1$

$\boldsymbol{B} = \mu_o n I^{/}$, where $I^{/} = 2.0A$

$\Rightarrow \boldsymbol{B} = 4\pi \times 10^{-7} \times 750 \times 2$

$\therefore \boldsymbol{B} = 1.884956 \times 10^{-3}\boldsymbol{T}$

$\boldsymbol{A} = \pi \times 0.025^2 = 1.963495 \times 10^{-3}\boldsymbol{m}^2$

and $\boldsymbol{I} = 1.0A$

$\tau = 1.884956 \times 10^{-3} \times 1 \times 10 \times 1.963495 \times 10^{-3}$

$\therefore \boldsymbol{Torque, \tau} = 3.701102 \times 10^{-5}\ \boldsymbol{Nm}$

6. A circular coil of 20 turns each of radius 10.0 cm lies on a flat table. The earth's magnetic field intensity at the location of the coil is $43.8\boldsymbol{Am}^{-1}$ while the angle of dip is 67.0^o. Find the:

(a) magnetic flux threading the coil.

(b) torque on the coil if a current of 2.0***A*** is passed through it.

Solution

(a) Flux $\phi = AB\sin 67.0^o$,where

$B = \mu_o H = 4\pi \times 10^{-7} \times 43.8$

$\Rightarrow B = 5.504070 \times 10^{-5} T$,

$A = \pi \times 0.1^2 = 3.141593 \times 10^{-2} m^2$ and

67.0^o is the angle between the resultant field and the plane of the coil.

$\phi = 0.03141593 \times 5.50407 \times 10^{-5} \times \sin 67.0^o$

$\therefore \phi = 1.591695 \times 10^{-6} Tm^2$

(b) Using $Torque, \tau = BINA\cos\alpha$;

where $\alpha = 67.0^o$ and $I = 2.0A$

$\tau = 5.50407 \times 10^{-5} \times 2 \times 20$

$\times 3.141593 \times 10^{-2} \times \sin 67^o$

$\therefore Torque, \tau = 2.702538 \times 10^{-5} Nm$

Trial questions

1. A vertical rectangular coil is suspended from the middle of is upper side with its plane parallel to the uniform horizontal magnetic field of 0.06T. the coil has 50 turns and the length of its vertical and horizontal sides are 4cm and 5cm respectively. Find the torque on the coil when the current of 4A is passed through it. (***Ans*** 0.024***Nm***)

3. A circular coil of 20 turns each of radius 10 cm is suspended with its plane along uniform magnetic field of flux density 0.5***T***. find the initial torque on the coil when a current of 1.5 ***A*** is passed through it. (***Ans*** 0.471239***Nm***)

4. A vertical square coil of sides 25 cm has 100 turns and carries a current of 1***A***. Calculate the torque on the coil when it is placed in a horizontal magnetic field of flux density 0.2***T*** with its plane making an angle of 30^o to the field. (***Ans*** 1.082532***Nm***)

The moving coil galvanometer.

Construction

It consists of a rectangular coil of fine insulated copper wire suspended in a strong magnetic field provided by curved pole pieces of a strong magnet.

The coil is wound on an aluminium frame (former) *to make it rest quickly when deflected due to electromagnetic damping due to* ***eddy currents*** *in the former.*

A soft iron cylinder between the curved pole pieces of a permanent magnet is *used to provide a radial field so that the field lines are always parallel to the plane of the coil whatever the deflection.*
This means that the magnetic flux density is interacting with the coil is constant and forces on the vertical sides of the coil are always perpendicular to the plane of the coil and the deflection torque has a maximum value for all positions.

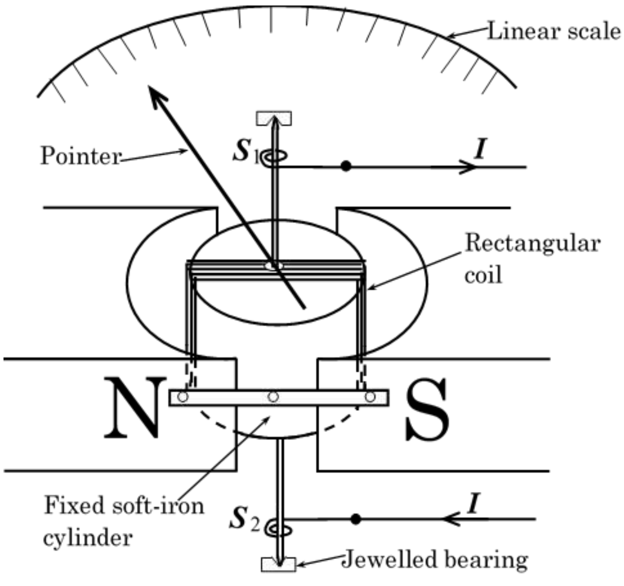
The coil is pivoted on jewelled bearings *to reduce friction.*

The current is led in and out through the hair-springs which then *control the coil rotation hence the pointer by providing a restoring couple.*

NB:

The magnetic field is made radial so as *to provide a linear scale in which the plane of the coil in all position remains parallel to the direction of the magnetic field.*

Diagram.



N and S are *curved pole pieces of a permanent magnet*,
S_1 and S_2 are *hair springs and* I is *current.*

Action

The current I to be measured is fed into the coil through the hair spring.
The coil experiences a magnetic torque, $\tau_m = BINA$ (deflecting torque).
The coil then turns in the magnetic field with the pointer until it is stopped by the restoring torque, $\tau_r = k\theta$ (opposing torque) provided by the hair spring, where k is the torsional constant of the hair spring and θ is the deflection of the pointer.
At the point of no deflection, the magnetic torque is equal to the restoring torque, that is

$BINA = k\theta . \Rightarrow I = \left(\frac{k}{BAN}\right)\theta$. Hence,

$I \propto \theta$, since $\left(\frac{k}{BAN}\right)$ is a constant and so the reading is taken on a linear scale.

Sensitivity of an ammeter / Current sensitivity.

The sensitivity of a current refers to the deflection per unit current.

From, $I = \left(\frac{k}{BAN}\right)\theta$,

Current sensitivity, $\frac{\theta}{I} = \frac{BAN}{k}$.

NB:

(i) Greater values of B increase current sensitivity.
(ii) Low value of k (weak springs) increases current sensitivity.
(iii) Greater values of N and A also increase current sensitivity.

Sensitivity of a voltmeter / Voltage sensitivity.

This is the deflection per unit potential difference.

Voltage sensitivity $= \frac{\theta}{V}$, where θ is the deflection produced by a p.d, V.

If the resistance of the moving coil meter is R, the p.d, V across its terminals when a current, I flows through is given by,

$$V = IR \Rightarrow I = \frac{V}{R}.$$

From, $\frac{V}{R} = \left(\frac{k}{BNA}\right)\theta$

Voltage sensitivity, $\frac{\theta}{V} = \frac{BNA}{Rk}$

NB:

(i) Greater values of B increase the voltage sensitivity.
(ii) Greater values of N and A also increase voltage sensitivity.
(iii) Low value of k (weak springs) increases voltage sensitivity.
(iv) Low resistance value of the coil increases voltage sensitivity.

Note

If the field is not radial, that is, the plane of the coil is at an angle α to the field, then deflecting torque, $\tau_m = BINA\cos\alpha$
opposing torque, $\tau_r = k\theta$
At a point of no deflection,

$$\Rightarrow BINA\cos\alpha = k\theta$$

$I\left(\frac{BAN}{k}\right) = \theta\sec\alpha$, giving a **Non**-linear scale on the galvanometer.

NB

Advantages of a moving coil galvanometer.

(i) Since the scale is uniform, the calibration can be made uniform and subdivisions read accurately.
(ii) It has a linear scale because of the uniform field provided by the radial field.
(iii) It can be made to measure different ranges of current and potential difference.

(iv) External field around the galvanometer has no influence because the magnetic field between the magnets and the soft iron is very strong.

Disadvantage of a moving coil galvanometer.

- ✓ If it is insensitive, it gives inaccurate results.
- ✓ It can not measure alternating current (A.C).
- ✓ It can be damped by overloading and so, the springs burn out.

NUMERICAL EXAMPLES

1. A rectangular coil of 100 turns is suspended in uniform magnetic field of flux density 0.02$\boldsymbol{T}$ with the plane of the coil parallel to the field. The coil is 3 cm high and 2 cm wide. If a current of 50$\boldsymbol{A}$ through the coil causes a deflection 30^{o}, calculate the torsional constant of the suspension.

Solution

At equilibrium (Point of no deflection),

$$\boldsymbol{BINA = k\theta \Rightarrow k = \frac{BINA}{\theta}}, \text{ where}$$

$$\boldsymbol{\theta} = 30^{o} = \frac{30\pi}{180} = \frac{\pi}{6}\,\boldsymbol{rads} \text{ and}$$

$$\boldsymbol{A} = \frac{3\times 2}{10000} = 6.0\times 10^{-4}\,\boldsymbol{m}^{2}$$

$$\Rightarrow \boldsymbol{k} = \frac{0.02\times 50\times 100\times 6.0\times 10^{-4}}{\frac{\pi}{6}}$$

$$\therefore \boldsymbol{k} = 0.114592\,\boldsymbol{Nm\ rad}^{-1}$$

2. A moving coil galvanometer has a rectangular coil of 6cm ***by*** 2 cm and 40 turns. It is suspended with its longer side vertical in a radial magnetic field of flux density 0.5$\boldsymbol{T}$ by means of glass fibre which produces a restoring torque of $0.35\,\boldsymbol{Nm\ rad}^{-1}$. What is the deflection produced when a current of 12$\boldsymbol{A}$ passes through it?

Solution

At equilibrium (Point of no deflection),

$$\boldsymbol{BINA = k\theta \Rightarrow \theta = \frac{BINA}{k}}, \text{ where}$$

$$\boldsymbol{A} = \frac{6\times 2}{10000} = 1.2\times 10^{-3}\,\boldsymbol{m}^{2} \text{ and}$$

$$\boldsymbol{k} = 0.35\,\boldsymbol{Nm\ rad}^{-1}$$

$$\Rightarrow \boldsymbol{\theta} = \frac{0.5\times 12\times 40\times 1.2\times 10^{-3}}{0.35}$$

$$\therefore \boldsymbol{k} = 0.822857\,\boldsymbol{rads} \approx 47^{o}.$$

<u>Trial questions</u>

1. The rectangular coil of a moving coil galvanometer of 100 turns is suspended in a uniform magnetic field of flux density 0.02$\boldsymbol{T}$ with the plane of the coil parallel to the field. The coil is 6 cm ***by*** 2cm. When a current of 50 $\boldsymbol{\mu A}$ is passed through the coil, the deflection of the point goes through 30^{o}. Calculate the torsional constant of the suspension.
($\boldsymbol{Ans}$ $2.2918311\times 10^{-7}\,\boldsymbol{Nm\ rad}^{-1}$)

2. The coil in a certain galvanometer is rectangular with sides 4 cm ***by*** 3 cm and with 150 turns. Calculate the initial deflecting moment of a couple due to the current of 4$\boldsymbol{mA}$ if the magnetic flux density is 0.02$\boldsymbol{T}$.
($\boldsymbol{Ans}$ $1.440\times 10^{-5}\,\boldsymbol{Nm}$)

3. The moving coil galvanometer has the following parameters. $\boldsymbol{N} = 180\,\boldsymbol{turns}$, $\boldsymbol{A} = 80\,\boldsymbol{mm}^{2}$, $\boldsymbol{B} = 0.2\boldsymbol{T}$, $\boldsymbol{R} = 120\Omega$, and $\boldsymbol{k} = 15\times 10^{-9}\,\boldsymbol{Nm\ rad}^{-1}$. Calculate the angular deflection in degrees, produced by

(i) a current of 0.01$\boldsymbol{mA}$.
($\boldsymbol{Ans}$ 110^{o})

(ii) a P.D of 0.05$\boldsymbol{mV}$.
($\boldsymbol{Ans}$ 4.6^{o})

CONVERSION OF A GALVANOMETER TO AN AMMETER OR VOLTMETER.

The deflection of the pointer on a moving coil galvanometer depends on the current flowing through the coil.

The current that produces full deflection is called ***full scale deflection current***, I_f.

The coil offers some resistance, r, to the flow of current through it due to its length. This does not change as current flows through the meter.

The ***full scale deflection voltage***, V_f is given by $V_f = I_f \times r$

Example

A meter has a full scale deflection voltage of 100 *mV* and full scale deflection current of 10 *mA*. What is the meter resistance?

Solution

$V_f = I_f \times r$; where

$V_f = 100mV = 0.1V, I_f = 10mA = 0.01A$

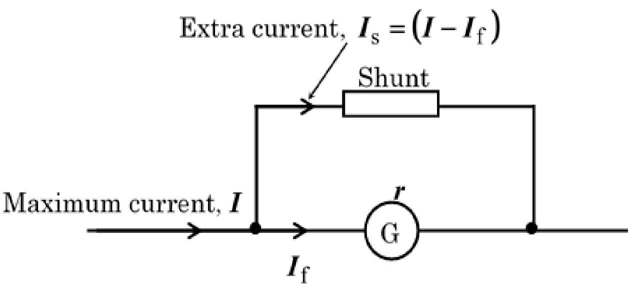
$\Rightarrow 0.1 = 0.01 \times r \quad \therefore \quad r = 10\Omega$.

(i) Conversion of a galvanometer to an ammeter (Use of shunts)

An ammeter is constructed in such a way that it has a very low resistance so that a large current passes through it.

To convert a galvanometer into an ammeter, a low resistance called a shunt is connected in parallel with it.

In this case, only maximum current ***full scale deflection current***, I_f flows through the meter and the rest of the current by-passes the meter through the shunt.



If I is the maximum current to be measured, I_s is the current through the shunt and I_f is the full scale deflection current, then

$$I_s = (I - I_f)$$

P.d across the shunt = P.d across the galvanometer

$\Rightarrow \quad (I - I_f) \times R_s = I_f \times r$

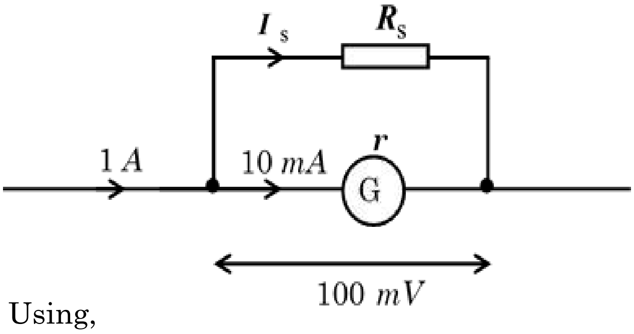
Where, R_s is the shunt resistance which small such that most of the current pass through it and only a small current pass through the galvanometer.

Examples:

1. A 0-10 *mA* meter has a full-scale deflection when the potential difference across it is 100 *mV*. How would you adapt the meter to read 0-1A?

Solution

In this case, we need to calculate the value of R_s that can make it possible



Using,

P.d across the shunt = P.d across the G

$\Rightarrow \quad (I - I_f) \times R_s = I_f \times r$ and thus,

$\Rightarrow \quad (1 - 0.01) \times R_s = (I_f \times r) = 0.1$

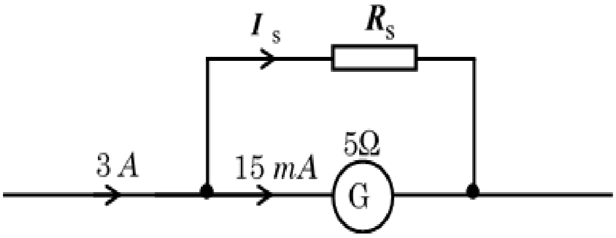
$$\therefore \quad R_s = \frac{0.1}{0.99} = 0.10\Omega$$

Thus, we must connect a shunt resistance of 0.10 Ω across the meter.

2. A moving coil galvanometer has a resistance of 5Ω and gives a full deflection of 15mA. How can it be converted into an ammeter to measure a maximum of 3A?

Solution

In this case, we need to calculate the value of R_S that can make it possible



Using,

P.d across the shunt = P.d across the galvanometer

$\Rightarrow \quad (I - I_f) \times R_S = I_f \times r$ and thus,

$$\Rightarrow \quad (3 - 0.015) \times R_S = (0.015 \times 5)$$

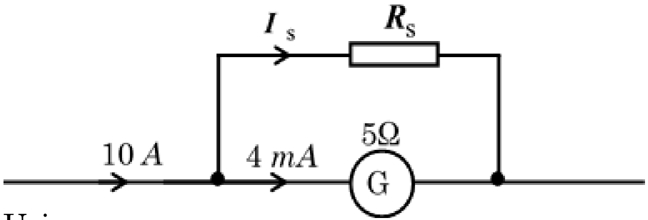
$$\therefore \quad R_S = \frac{0.075}{2.985} = 0.02513\Omega$$

Thus, we must connect a shunt resistance of 0.02513 Ω across the meter.

3. A moving coil galvanometer gives a full scale deflection of 4mA and has a resistance of 5Ω. How can such instrument be converted into an ammeter giving a full-scale deflection of 10A?

Solution

In this case, we need to calculate the value of R_S that can make it possible



Using,

P.d across the shunt = P.d across the galvanometer

$\Rightarrow \quad (I - I_f) \times R_S = I_f \times r$ and thus,

$$\Rightarrow \quad (10 - 0.004) \times R_S = (0.004 \times 5)$$

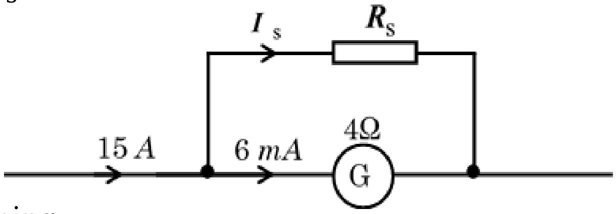
$$\therefore \quad R_S = \frac{0.02}{9.996} = 2.0 \times 10^{-3}\Omega$$

Thus, we must connect a shunt resistance of $2.0 \times 10^{-3}\Omega$ across the meter.

4. A moving coil galvanometer gives a full scale deflection of 6mA and has a resistance of 4Ω. How can such instrument be converted into an ammeter giving a full-scale deflection of 15A?

Solution

In this case, we need to calculate the value of R_S that can make it possible



Using,

P.d across the shunt = P.d across the galvanometer

$\Rightarrow \quad (I - I_f) \times R_S = I_f \times r$ and thus,

$$\Rightarrow \quad (15 - 0.006) \times R_S = (0.006 \times 4)$$

$$\therefore \quad R_S = \frac{0.024}{14.996} = 1.6 \times 10^{-3}\Omega$$

Thus, we must connect a shunt resistance of $1.6 \times 10^{-3}\Omega$ across the meter.

Exercise

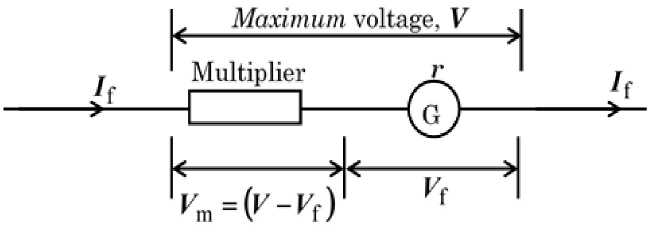
1. A moving coil galvanometer of resistance 5Ω and current sensitivity of 2 *divisions per milliampere*, gives a full-scale deflection of 16 *divisions*. Explain how such an instrument can be converted into an ammeter reading up to 20*A* (*Current sensitivity* $= \frac{\text{Number of divisios}}{\text{Current}}$)
2. A moving coil galvanometer, has a coil of resistance 4Ω and gives a full scale deflection when a current of 25 *mA* passes through it. Calculate the value of the resistance required to convert it to an ammeter which reads 15*A* at full scale deflection.
3. A galvanometer has a resistance of 5Ω and range 0-40 *mA*. Find the resistance of the resistor which must be connected in parallel with the galvanometer if a maximum current of 10*A* is to be measured.

4. A moving coil galvanometer has a resistance of 25Ω and gives a full scale deflection of when carrying a current of 4.4 μA. What current will give a full-scale deflection when the galvanometer is shunted by a 0.10Ω resistance?

$(Ans\, 1.0956 \times 10^{-3} A)$

(ii) Conversion of a galvanometer to a voltmeter (Use of multipliers)

A voltmeter has a high resistance so that no current passes through it.
To convert a galvanometer to a voltmeter, a high resistance called a multiplier is connected in series with it.
In this case, only maximum voltage, ***full scale deflection voltage***, V_f drops across the meter and the rest of the potential difference drops across the multiplier.



If V is the maximum voltage to be measured, R_m is the voltagge across the multiplier and I_f is the current at full scale deflection voltage, then

$$V_m = (V - V_f)$$

Current, I_f through M = Current, I_f through G

$$\Rightarrow \quad \frac{V_m}{R_m} = \frac{V_f}{r} \quad \therefore \quad \frac{(V - V_f)}{R_m} = \frac{V_f}{r}$$

M is multiplier and G is a galvanometer

Note:

P.d across M $(V - V_f)$ = Current, $I_f \times R_m$

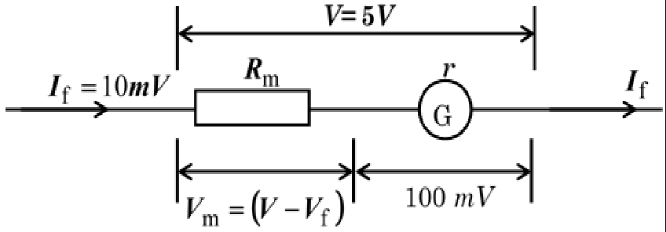
P.d across G V_f = Current, $I_f \times r$

Examples

1. A 0-10 mA meter has a full-scale deflection when the potential difference across it is 100 mV. How would you adapt the meter to read 0-5V?

Solution

In this case, we need to calculate the value of R_m that can make it possible



Using,

P.d across the multiplier $(V - V_f)$

$= \text{Current, } I_f \times R_m$

$\Rightarrow \quad (5 - 0.1) = 0.01 \times R_m$ and thus

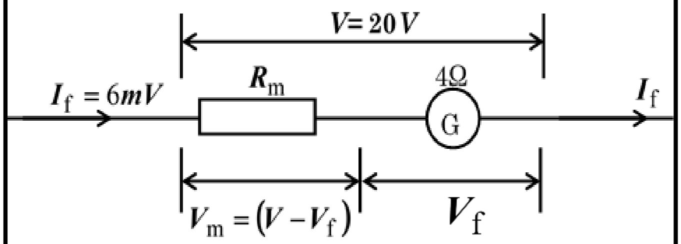
$$\therefore \quad R_m = \frac{4.9}{0.01} = 490\Omega$$

Thus, we must connect a multiplier resistance of 490Ω in series with the meter.

2. A moving coil galvanometer gives a full scale deflection of 6 mA and has a resistance of 4Ω. How can such instrument be converted into a voltmeter reading up to 20V?

Solution

In this case, we need to calculate the value of R_m that can make it possible



Using,

P.d across M, $(V - V_f)$ = Current, $I_f \times R_m$

But, $V_f = 0.006 \times 4 = 0.024V$

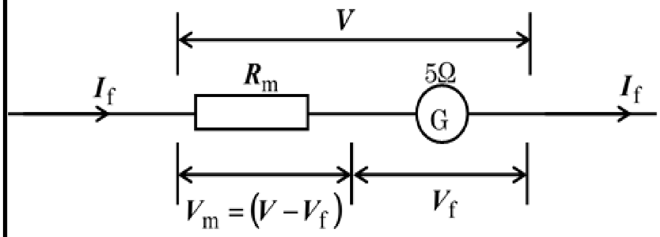
$\Rightarrow \quad (20 - 0.024) = 0.006 \times R_m$

$\therefore \quad R_m = \dfrac{19.976}{0.006} = 3329.33\Omega$. Thus, we must connect a multiplier resistance of 3329.33Ω in series with the meter.

3. A moving coil galvanometer of resistance 5Ω and current sensitivity of 2 *divisions per milliampere*, gives a full-scale deflection of 16 *divisions*. Explain how such an instrument can be converted into a voltmeter in which each division represents $2V$?

Solution

In this case, we need to calculate the value of R_m that can make it possible



Using,

$$\text{P.d across the multiplier } (V - V_f)$$

$$= \text{Current, } I_f \times R_m$$

But

$$\text{Current sensitivity, } K_I = \frac{\text{Number of divisions}}{I_f}$$

$$\Rightarrow \quad 2divs/mA = \frac{16divs}{I_f} \quad \therefore I_f = 8mA$$

$$\Rightarrow \quad V_f = 0.008 \times 5 = 0.04V$$, Also,

$$1div = 2V \quad \Rightarrow 16divs = 16 \times 2 = 32V \quad \therefore V = 32V$$

$$\Rightarrow \quad (32 - 0.04) = 0.008 \times R_m$$

$$\therefore \quad R_m = \frac{31.96}{0.008} = 3995\Omega.$$

Thus, we must connect a multiplier resistance of $3995\,\Omega$ in series with the meter.

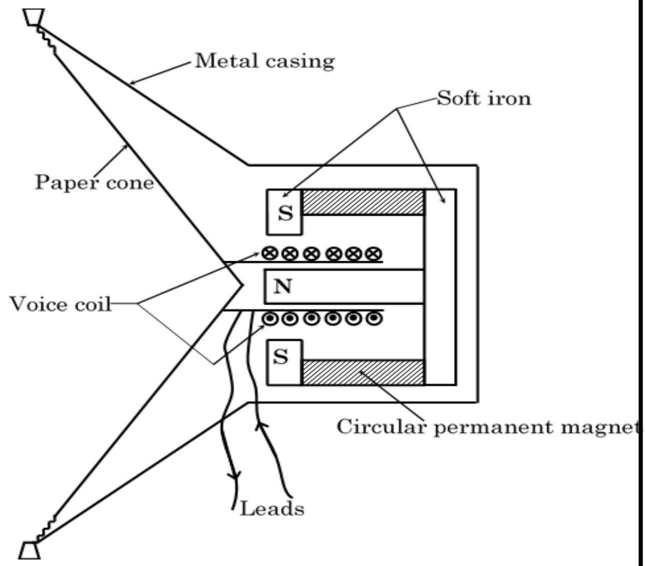
Exercise

1. A galvanometer of reads $0.05A$ at full scale deflection and has resistance of 2.0Ω. Calculate the resistance that should be connected in series with it to convert it to a voltmeter which reads $15V$ at full scale deflection.
2. A galvanometer of internal resistance100 Ω gives full-scale deflection of 10 *mA*. Calculate the value of the resistance necessary to convert it to voltmeter reading up to $5V$.

3. A galvanometer of resistance 12Ω reads 200 *mA* at full-scale deflection. what resistance must be connected in series with it in order to read $8V$.

The Moving coil loud speaker.

Structure

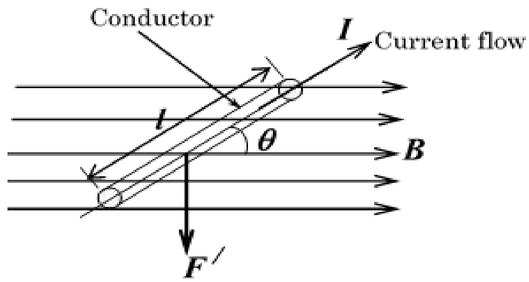


Action

- ✓ An alternating (varying) current from an amplifier flows through the voice coil and produces a changing magnetic field around it. The coil is in the magnetic field of the circular permanent magnet.
- ✓ The interaction of the two magnetic fields sets up a varying force on the voice coil according to Fleming's left hand rule.
- ✓ The varying force therefore in turn causes the vibration of the voice coil.
- ✓ The paper cone to which the voice coil is connected is also set into vibrations.
- ✓ This causes the air molecules around the paper cone also to vibrate, hence producing a sound of the same pattern as the original electrical signal sent into the coil.
- ✓

Force on charges moving in a magnetic field.

Consider a conducting wire of length, l, containing N charged particles placed in a uniform magnetic field of flux density, B as shown below.



Let q be the charge carried by each charged particle moving in the conductor at an average drift velocity, V.

The time taken by every particle to move through the length, l is given by

$$V = \frac{l}{t}$$

Total charge passing through any cross-section area of the conductor in time, t is

$$Q = Nq$$

Current, I flowing is given by;

$$I = \frac{Nq}{t} = \frac{NVq}{l}$$

The force, $F^{/}$ on a wire of length, l in a magnetic field is given by,

$$F^{/} = BIl \sin\theta,$$

$$\Rightarrow \quad F^{/} = B\left(\frac{NVq}{l}\right) l \sin\theta$$

$$= BNqV \sin\theta$$

Therefore, the force one charged particle is

$$F = BqV \sin\theta$$

Note

(i) **Generally,** the expression of the magnetic force on a charge, Q moving in a conductor inclined at an angle, θ to the uniform field of flux density, B is given by,

$$F = BQv\sin\theta.$$

(ii) If the charge is moving perpendicular to the magnetic field of flux density, B, $\theta = 0^{o}$ then, the magnetic force is,

$$F = BQv.$$

NUMERICAL EXAMPLES

1. The force acting on charge Q flowing through a conductor placed in a magnetic field is given by $F = BQV\sin\theta$. Deduce an expression for the maximum force acting on the conductor carrying current, I and placed in the magnetic field of flux density, B, if l is the length of the conductor and V is an average drift velocity of each charged particle.

Solution.

If q is the charge carried by each charged particle and N is the number of the charged particles in the conductor, then

$$Q = Nq$$

But , $V = \frac{l}{t}$, where t is the time taken for the particles to move through length, l.

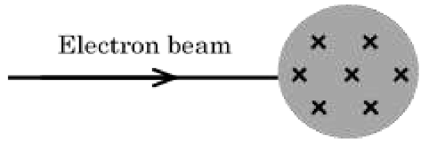
$$F = BNq\left(\frac{l}{t}\right) \sin\theta.$$

But also, $Nq = It$ and $\theta = 90^{o}$ for maximum force on the conductor.

$$\Rightarrow F_{max} = BIt\left(\frac{l}{t}\right) \sin 90^{o}$$

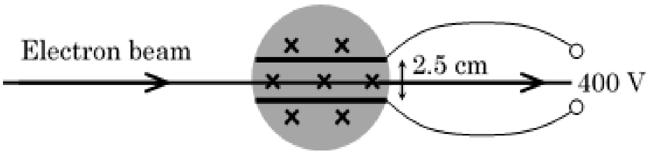
$$\therefore F_{max} = BIl.$$

2.



(a) The figure above shows a beam of electrons accelerated by a potential difference, V, travelling in an evacuated tube. A magnetic field acts at right angles to their direction of motion in the shaded region and into the plane of the paper. Copy the diagram and complete the path of the electrons in the shaded region and explain why the electron beam takes the indicated path.

(b) A pair of conducting plates, 2.5 cm apart, has been introduced into the shaded region.



A p.d is applied to the plates and gradually increased until it reaches $400\,V$ when the path of the electrons is a straight line.

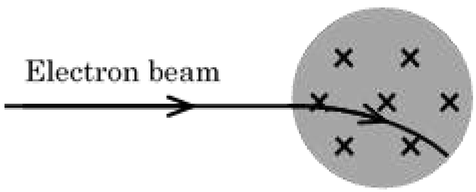
(i) Indicate the polarity of the plates and briefly explain why the plates bear the polarities indicated.
(ii) Determine the electric field strength in the region between the plates
(iii) Calculate the force on an electron due to this field.

(c) The magnetic flux density in the shaded region is $1.0\times10^{-3}\,T$. Calculate the

(i) speed of the electrons for them to move straight through the field.
(ii) p.d required to accelerate the electrons at this speed.

Solution.

(a)

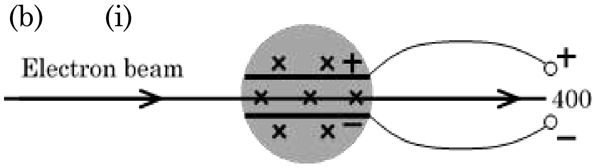


An electron moves in a conductor in the direction opposite to that of current. The current therefore is flowing to the left hand side. According to Fleming's left hand rule, the force acts on the beam downwards, making it to bend downward but in a circular path since the beam possesses kinetic energy.

NOTE

When the electrons exit the magnetic field, they move in a straight line which is tangential to the circular path at the point of exit

(b) (i)



Since the electrons carry a negative charge, then for them to be deflected upwards, then a positive potential must be applied to the upper plate and a negative charge to the lower plate.

(ii) Using $E = \frac{V}{d}$

$$\Rightarrow E = \frac{400}{2.5\times10^{-2}} = 1.6\times10^{4}\,Vm^{-1}.$$

(iii) By definition of electric field intensity,

$$F = eE$$

$$\Rightarrow F = 1.6\times10^{-19}\times1.6\times10^{4}\,Vm^{-1}$$

$$\therefore F = 2.56\times10^{-15}\,N$$

(c) (i) From $F = BeV\sin\theta$

$\theta = 90^{o} \Rightarrow \sin\theta = \sin 90^{o} = 1$ and

$B = 1.0\times10^{-3}\,T$

$$\Rightarrow 2.56\times10^{-15} = 1\times10^{-3}\times1.6\times10^{-19}V_e$$

$$\therefore\ V_e = 1.6\times10^{7}\,ms^{-1}.$$

(ii) Electrical work done on the electron by the field = Kinetic energy of the electron

$$\Rightarrow eV = \frac{1}{2}m_eV_e^{2}$$

$$\Rightarrow 1.6\times10^{-19}V = \frac{9.11\times10^{-31}\times\left(1.6\times10^{7}\right)^{2}}{2}$$

$$\therefore\quad V = 728.8\,V.$$

3. A metal rod of length 50 cm moves with a velocity of $5ms^{-1}$ in a plane normal to the magnetic field of flux density $0.05\boldsymbol{T}$. Find the

(i) magnetic force on the electron in the rod.
(ii) Electric field intensity in the rod.
(iii) p.d across the ends of the rod.

Solution

(i) From $\boldsymbol{F = BeV\ sin\theta}$

$\boldsymbol{\theta} = 90^{\boldsymbol{o}} \Rightarrow \boldsymbol{sin\theta} = \boldsymbol{sin}90^{\boldsymbol{o}} = 1$ and $\boldsymbol{B} = 0.05\boldsymbol{T}$

$$\Rightarrow \boldsymbol{F} = 0.05 \times 1.6 \times 10^{-19} \times 5 \times 1$$

$$\therefore \boldsymbol{F} = 4.0 \times 10^{-20}\boldsymbol{N}.$$

(ii) By definition of electric field intensity,

$$\boldsymbol{F} = \boldsymbol{eE}$$

$$\Rightarrow 4.0 \times 10^{-20} = 1.6 \times 10^{-19}\boldsymbol{E}$$

$$\therefore \boldsymbol{E} = 0.25\boldsymbol{Vm}^{-1}.$$

(iii) Using $\boldsymbol{E} = \frac{\boldsymbol{V}}{\boldsymbol{d}} \Rightarrow 0.25 = \frac{\boldsymbol{V}}{50 \times 10^{-2}}$

$$\therefore \boldsymbol{V} = 0.125\boldsymbol{V}.$$

4. The figure below shows a magnetic deflector placed in a vacuum, used to deflect protons into a proton detector using a magnetic field, which is uniform within the square and zero outside it.

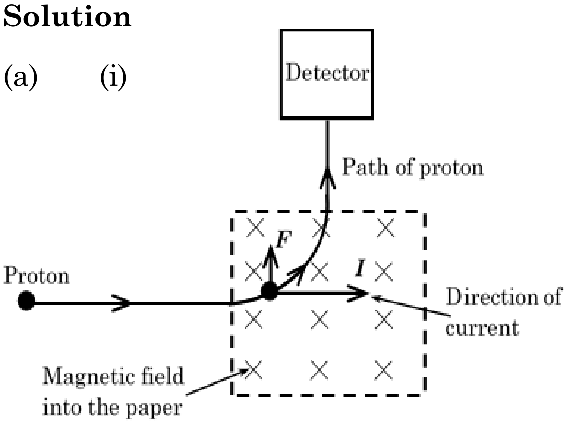
(a) (i) Copy the diagram and sketch the path of the proton in the magnetic field. On this path at any point show the direction of the magnetic force on it.
(ii) State the direction of the magnetic field causing the motion of the proton.

(b) (i) If the speed of the proton as it enters the magnetic field of flux density $0.5\boldsymbol{T}$ is $5.0 \times 10^6 \boldsymbol{ms}^{-1}$, calculate the magnitude of the magnetic force on the proton.

(ii) If the path were that of an electron with the same speed, briefly explain **two** changes that can be made on the magnetic field to make the electrons enter the detector along the same path.

Solution

(a) (i)



(ii) The magnetic field is into the paper according to Fleming's left hand rule.

(b) (i) From $\boldsymbol{F = BeV\ sin\theta}$

$\boldsymbol{\theta} = 90^{\boldsymbol{o}} \Rightarrow \boldsymbol{sin\theta} = \boldsymbol{sin}90^{\boldsymbol{o}} = 1$ and $\boldsymbol{B} = 0.5\boldsymbol{T}$

$$\Rightarrow \boldsymbol{F} = 0.5 \times 1.6 \times 10^{-19} \times 5 \times 10^6 \times 1$$

$$\therefore \boldsymbol{F} = 4.0 \times 10^{-13}\boldsymbol{N}.$$

(ii) The size of the flux density must be reduced since an electron is less massive than a proton and so, it undergoes a short horizontal deflection.

The direction of the magnetic field should be in such a way that it is out of the paper for the electron to be deflected upwards according to the motor rule since the current will be flowing to the left.

5. A copper wire of cross-sectional area $1.5\,mm^2$ carries a current of $5.0A$. The wire is placed perpendicular to a magnetic field of flux density $0.2T$. If the density of free electrons in the wire is $10^{29}m^{-3}$, calculate the force on each electron.

Solution

Using $F = BeV\sin\theta$, where

$\theta = 90^o \Rightarrow \sin\theta = \sin 90^o = 1, B = 0.2T$

and $V = \dfrac{I}{neA}$

$$\Rightarrow V = \frac{5.0}{10^{29}\times 1.6\times 10^{-19}\times 1.5\times 10^{-6}}$$

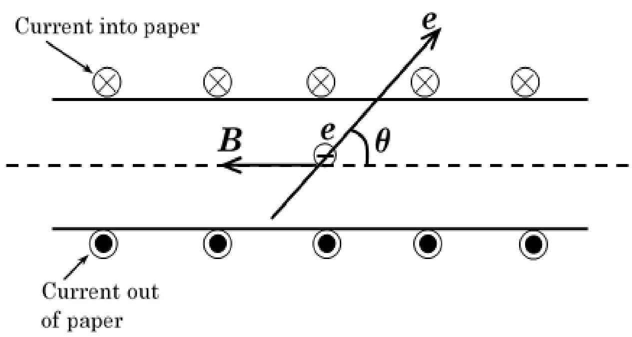
$\therefore V = 2.0833\times 10^{-4}ms^{-1}$

$F = 0.2\times 1.6\times 10^{-19}\times 2.0833\times 10^{-4}\times 1$

$\therefore\ F = 6.667\times 10^{-24}N$.

6. A current of $3.25A$ flows through a long solenoid of 400 turns and length 40.0 cm. Determine the magnitude of the force exerted on a particle of charge $15\mu C$ moving at $1.0\times 10^3 ms^{-1}$ through the centre of the solenoid at an angle of 11.5^o, relative to the axis of the solenoid.

Solution



Using $F = BeV\sin\theta$, where

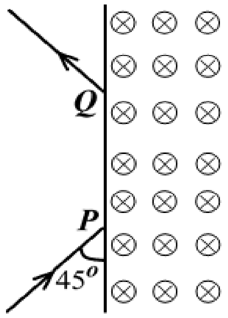
$\theta = 11.5^o, B = \mu_o nI$

$$\Rightarrow B = 4\pi\times\frac{400}{0.4}\times 3.25 = 4.08407\times 10^{-3}T$$

$F = 0.00408407\left(15\times 10^{-6}\times 10^3\right)\sin 11.5^o$

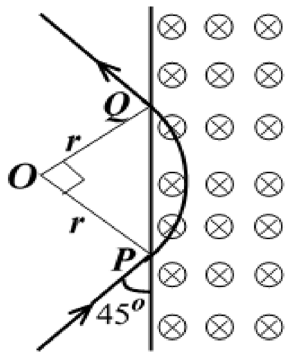
$\therefore F = 1.221349\times 10^{-5}N$.

7. A charged particle of mass $1.4\times 10^{27}kg$ and charge $1.6\times 10^{-19}C$, enters a region of a uniform magnetic field of flux density $0.2T$ at point P and emerges at point Q as shown below.



If the speed of the particle is $10^7 ms^{-1}$, calculate the distance PQ.

Solution



The magnetic force on the particle provides the necessary centripetal force that makes the particle to describe a circular path in the magnetic field.

Thus,

$$F = BeV = \frac{mV^2}{r} \Rightarrow r = \frac{mV}{Be}$$

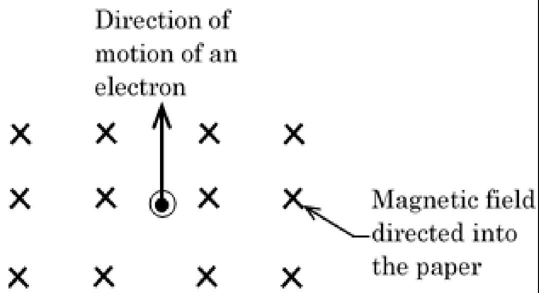
$$\Rightarrow r = \frac{1.4\times 10^{-27}\times 10^7}{0.2\times 1.6\times 10^{-19}} = 0.4375m$$

$$\Rightarrow PQ = \sqrt{0.4375^2 + 0.4375^2}$$

$\therefore PQ = 0.6187m$.

Trial question

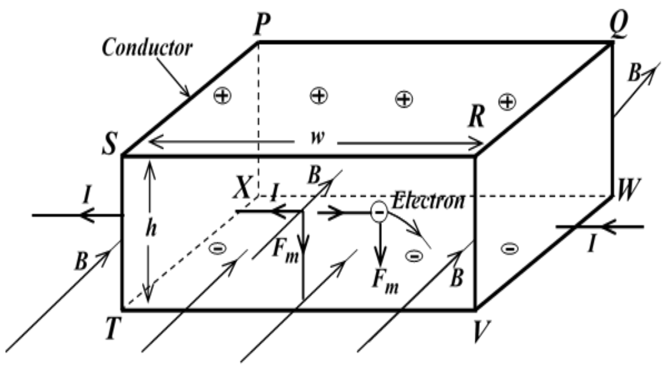
1. An electron beam moving with velocity of 100m/s passes through a uniform magnetic field of flux density 0.04T which is perpendicular to the direction of the beam. Calculate the fore on each electron. (***Ans*** $6.4\times10^{-19}\,\boldsymbol{N}$)
2. Determine the force on an electron that enters a uniform magnetic field of flux density 150 ***m T*** at a velocity of $8.0\times10^{6}\,\boldsymbol{ms}^{-1}$ at an angle of
   (i) 90^{o},
   (ii) 0^{o} to the field.
3. Electrons in a vertical wire move upwards at a speed of $2.5\times10^{3}\,\boldsymbol{ms}^{-1}$ into a uniform horizontal magnetic field of magnetic flux density 95 ***m T***. The field is directed along a line from south to north as shown in Figure below.
   Calculate the force on each electron and determine its direction.



Hall effect

This refers to the production of a p. d across an electrical conductor, transverse to an electric current in the conductor and to an applied magnetic field perpendicular to the current.

Explanation of its occurrence.



When current flows through the conductor ***PQRSTVWX*** from face ***QRVW*** towards face ***PSTX***, the electrons (charge carriers) in the conductor move in the opposite direction.

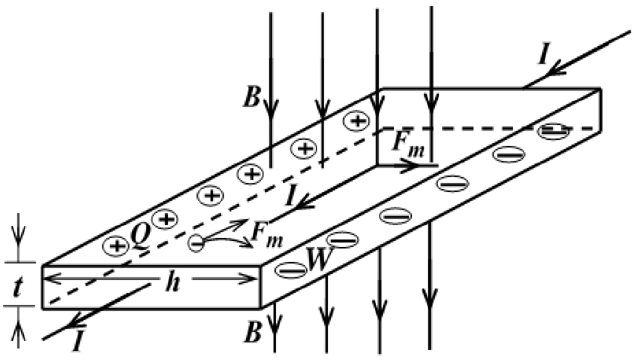
When a magnetic field, ***B*** is applied to the conductor from face ***RSTV*** towards face ***PQWX*** (say at right angles to the conductor) a force, $\boldsymbol{F = BeV}$ acts on the electron downwards towards the lower face, ***TVWX*** according to Fleming's left hand rule.

Therefore, the lower face gains a negative charge, leaving the upper face with a positive charge and so a p.d is created between the lower and upper face.

A large p.d, hence a large electric field builds up between the faces, preventing a further increase of charges on the faces. This maximum p.d is known as ***Hall voltage*** and the effect is called ***Hall effect***.

Derivation of hall voltage, (V_H)

Consider a current, ***I*** flowing through a conductor placed horizontally at right angles to the magnetic field of flux density, ***B***. Electrons (charge carriers flow through it in the opposite direction).



According to Fleming's left hand rule, a magnetic force, $\boldsymbol{F_m}$ acts on the negative charge carriers and push them to face ***W***, leaving positive charges on face, ***Q***. An electric field that builds up between faces ***W*** and ***Q*** opposes the further separation of the charge carriers. A maximum p.d called hall voltage, $\boldsymbol{V_H}$, is set up across the two faces.

At equilibrium;

$F_m = F_e$, where F_e is the electric field between faces W and Q.

$\Rightarrow BeV = eE$, since

$\theta = 90^o \Rightarrow sin\theta = sin90^o = 1$

But, $E = \frac{V_H}{h}$ $\Rightarrow V = \frac{E}{B} = \frac{V_H}{Bh}$.

Also, $V = \frac{I}{neA}; A = ht$ $\Rightarrow V = \frac{I}{neht}$.

Thus, $\frac{V_H}{Bh} = \frac{I}{neht}$ $\therefore V_H = \frac{BI}{net}$.

Note

1. Hall voltage is higher in semiconductors than in conductors because a semiconductor has fewer charge carriers than conductors and $V_H \propto \frac{1}{n}$.

2. Reducing the thickness, t of the material, increases hall voltage.

   Hall voltage can also be increased by increasing the current flowing through the conductor and also increasing the magnetic field normal to the conductor.

3. Hall effect can be used to

   * determine the sign of charge carrier flowing in a material.
   * determine the magnetic flux density using a hall probe.

Determination of sign of charge carriers in a conductor using hall effect.

* Current is passed through the material in a known direction and a magnetic field whose direction is also known is applied to the conductor at right angles to the direction of current.

* The direction of the magnetic force on the charge carriers is then determined using Fleming's left hand rule.

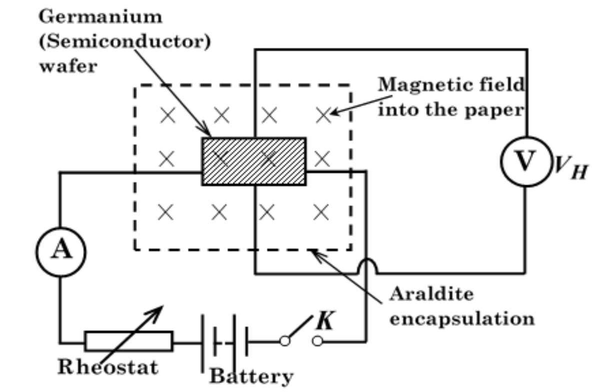
* The plate to which the charge carriers are pushed by the magnetic field, gain the charge of similar sign to the that on the charge carriers.

* The sign of charge is determined using a charged gold leaf electroscope.

* If the electroscope is positively charged and on connecting the plate the brass cap the deflection of the leaf increases, then the charge carriers bear a positive sign.

* If the electroscope is negatively charged and on connecting the plate the brass cap the deflection of the leaf increases, then the charge carriers bear a negative sign.

Determination of magnetic flux density at a point using hall effect.



Switch, K is closed and the rheostat is adjusted such that a suitable current, I flows through the wafer between its opposite faces.

The current is noted from an ammeter of full scale deflection of 1A.

A magnetic field whose flux density, B is to be determined is applied on the wafer with the wafer suitably at right angles to the field.

A hall voltage V_H is set up transversely across the wafer is noted from a high impedance voltage, V connected across the wafer.

From the theory of hall voltage, the flux density, B is calculated form

$$B = \frac{net}{I}V_H$$, where the value net is provided by the manufacturer of the wafer.

NB

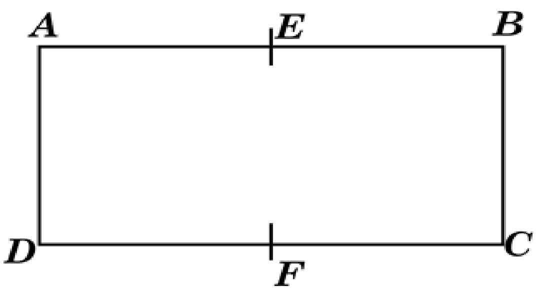
1. The Araldite encapsulation prevents the connecting wires from being detached from the wafer.

2. Other applications of hall effect include; -

- Magnetic field sensing equipment.
- Multiplier applications to provide actual multiplications.
- Hall Effect Tong Tester for measurement of direct current.
- Phase angle measurement, such as in measuring angular position of the crank shaft to accurately align with the firing angle of the spark plugs
- Linear or Angular displacement transducers, such as to identify the position of the car seats and seat belts and act as an interlock for air-bag control.
- For detecting wheel speed and accordingly assist anti-lock braking system (***ABS***).

NUMERICAL EXAMPLES

1. (a) ***ABCD*** is a plane rectangular strip of conducting material of uniform, with a steady current flowing uniformly from ***AD*** to ***BC***. The p.d across ***EF***, where E and F are mid-points of ***AB*** and ***CD*** respectively is zero.



When a magnetic field is set up at right angles to ***ABCD***, into the plane of the of the figure, a small, steady p.d appears between ***E*** and ***F***. Explain these observations.

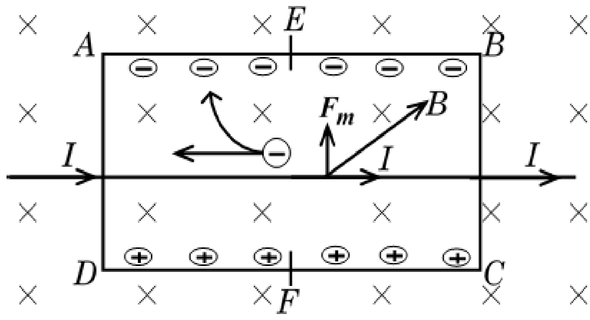
(b) If the strip is made of copper, of density $9\times10^{3}kgm^{-3}$, relative atomic mass 63, with thickness 1 mm, breadth 20 mm, carrying current of 10 A and with an

applied magnetic field of flux density 1.67***T***, the p.d across ***EF*** is found to be $1\mu V$, ***F*** being at a positive potential with respect to ***E***, find the:

(i) drift velocity of the current carriers in the strip.
(ii) number of charge carriers per volume in the copper and the sign of their charge.

Solution

(a)



When current flows from ***AD*** to ***BC***, electrons flow in the opposite direction.
When a magnetic field is applied at right angles to the strip, a magnetic force, F_m is set up in the strip and push the electrons to the side of ***E***, leaving positive charges on the side of ***F*** and this creates an electric field between ***E*** and ***F***, hence the appearance of a p.d across ***EF***.

(b) (i) Drift velocity, $V = \frac{E}{B}$, where

$$E = \frac{V_H}{h} = \frac{1.0\times10^{-6}}{20^{-3}} = 5.0\times10^{-5}Vm^{-1}$$

Thus, $V = \frac{5.0\times10^{-5}}{1.67} = 2.994012ms^{-1}$.

(ii) $B = \frac{net}{I}V_H \Rightarrow n = \frac{BI}{etV_H}$

$$\Rightarrow n = \frac{1.67\times10}{1.6\times10^{-19}\times1.0\times10^{-3}\times1.0\times10^{-6}}$$

$\therefore n = 1.04375\times10^{29}m^{-3}$.
The charge carriers have a negative sign since ***F*** is at a positive potential relative to ***E***.

2. A metallic strip of width 2.5 cm and thickness 0.5 cm carries a current of 10A. When a magnetic is applied normally to the broad side of the strip, a hall voltage of 2mV develops. Find the magnetic flux density if the conduction electron density is $6.0\times10^{28}\boldsymbol{m}^{-3}$.

Solution

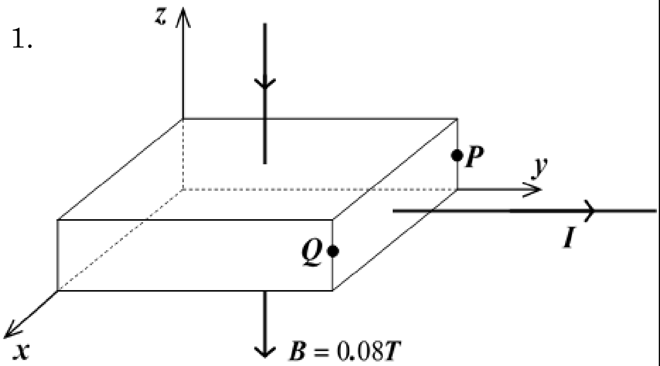
Using $\boldsymbol{B}=\dfrac{\boldsymbol{netV_H}}{\boldsymbol{I}}$

$$\boldsymbol{B}=\frac{6.0\times10^{28}\times1.6\times10^{-19}\times0.005\times0.002}{10}$$

$\therefore\ \boldsymbol{B}=9600\boldsymbol{T}$

Trial questions

1.



A magnetic flux density of 0.08$\boldsymbol{T}$ is applied normally to a metal strip carrying current, $\boldsymbol{I}$ as shown above.

(a) Account for the occurrence of a p.d between points $\boldsymbol{P}$ and $\boldsymbol{Q}$.

(b) Calculate the electric field intensity between P and Q if the drift velocity of the conduction electrons is $4.0\times10^{-4}\boldsymbol{ms}^{-1}$

2. A certain semi-conductor which is rectangular in shape with thickness of 1 mm is held in a uniform magnetic field parallel to the thickness of the semi-conductor. The magnetic flux density is 0.4$\boldsymbol{T}$. If the current passed through it is 2$\boldsymbol{A}$, the voltage developed is 0.28mV. Calculate the number of electrons per unit volume of the semi-conductor.

(***Ans*** $1.785714\times10^{25}\boldsymbol{m}^{-3}$)

3. A metal strip 120 mm wide and 0.4 mm thick carries a current of 2$\boldsymbol{A}$ at right angles to a uniform magnetic field of 2$\boldsymbol{T}$. If

the hall voltage is 4.27μV, calculate the drift velocity of the electrons in the strip and volume charge density of the carriers in the strip. (***Ans***, $\boldsymbol{V}=1.7791667\times10^{-5}\boldsymbol{ms}^{-1}$, $\boldsymbol{n}=1.463700\times10^{28}\boldsymbol{m}^{-3}$)

Force between two wires carrying currents.

When two wires carrying currents either in the same direction or opposite directions, they exert a magnetic force on each other.

The force between the wires can be repulsive or attractive force depending on the direction of the currents in the wires.

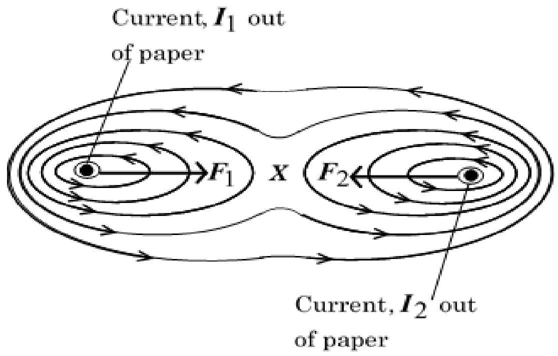
The force experienced is because each wire is placed in the magnetic field of the other.

NB

A wire placed in the earth's magnetic force also experiences the same kind of force on it.

Wires currying currents in the same direction.

Two straight conductors (wires) carrying currents, $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$, out of the paper. The resulting field pattern is as shown below.

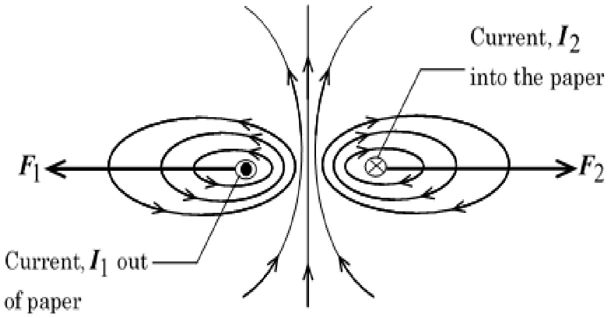


The force on each wire acts from the region of strong field to a region of a weak field. Thus, two straight parallel wires carrying current in the same direction attract each other.

$\boldsymbol{X}$ is a magnetic neutral point. At this point, the magnetic flux cancels out and the magnetic flux density is equal to zero and so, no magnetic force is experienced at this point.

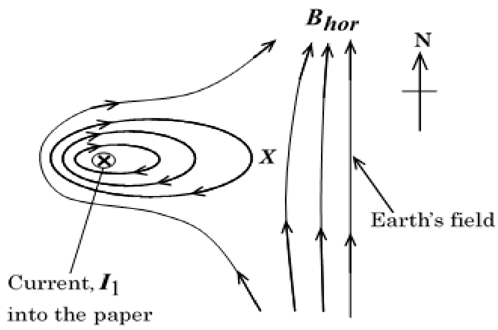
Wires currying currents in the opposite directions.

If the directions of currents I_1 and I_2, are opposite, the resultant magnetic field pattern is as shown below.

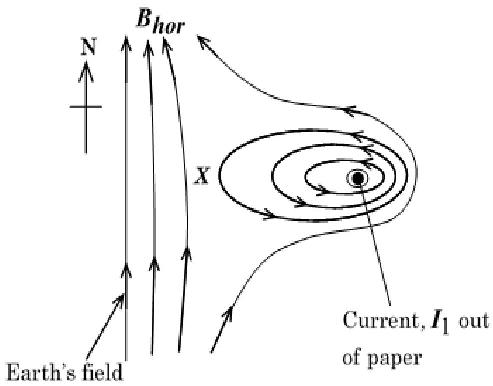


Thus, two parallel wires carrying currents in opposite directions repel each other.

Wire carrying current in the earth's horizontal magnetic field.

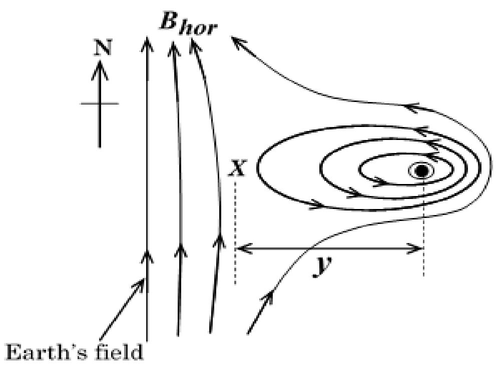


OR



NB

From the diagram below,



At point X (neutral point),

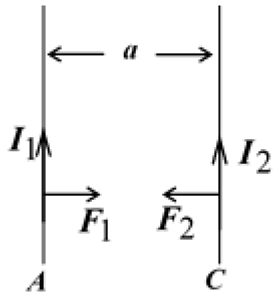
$$B_{hor} = B_1\text{, where, } B_1 = \frac{\mu_o I_1}{2\pi y}$$

$$\Rightarrow B_{hor} = \frac{\mu_o I_1}{2\pi y} \quad \therefore \; y = \frac{\mu_o I_1}{2\pi B_{hor}}$$

Thus, if the horizontal component, B_{hor} is known, the position, y of the neutral point from the wire can be obtained.

Derivation of expression of the force between currents.

Consider two parallel wires carrying currents I_1 and I_2 separated by a distance, a as shown below.



The magnitude of the magnetic flux density at C due to I_1 is given by, $B_1 = \frac{\mu_0 I_1}{2\pi a}$.

Thus, the force on C is given by,

$$F_2^{/} = B_1 I_2 l \quad \Rightarrow \quad F_2^{/} = \frac{\mu_0 I_1 I_2 l}{2\pi a}.$$

Hence, force per unit length on C is

$$F_2 = \frac{F_2^{/}}{l} \text{ and therefore, } F_2 = \frac{\mu_0 I_1 I_2}{2\pi a}.$$

Similarly;

The magnitude of the magnetic flux density at A due to I_2 is given by, $B_2 = \frac{\mu_0 I_2}{2\pi a}$.

Thus, the force on A is given by,

$$F_1^{/} = B_2 I_1 l \quad \Rightarrow \quad F_1^{/} = \frac{\mu_0 I_1 I_2 l}{2\pi a}.$$ Hence,

force per unit length on A is $F_1 = \frac{F_1^{/}}{l}$ and

therefore, $F_1 = \frac{\mu_0 I_1 I_2}{2\pi a}$.

Thus, when $I_1 = I_2 = 1A$ and a = 1 m in a vacuum then,

$$F_1 = \frac{4\pi \times 10^{-7} \times 1 \times 1}{2\pi \times 1} = 2.0 \times 10^{-7}\ Nm^{-1}$$

Also, $F_1 = 2.0 \times 10^{-7}\ Nm^{-1}$

Definition;

An **Ampere** refers to a steady current which when maintained in each of two straight parallel conductors of infinite length and negligible cross-sectional area separated by **1 m** length in vacuum produces between the conductors a force of $2.0 \times 10^{-7}\ Nm^{-1}$ length of each.

NUMERICAL EXAMPLES

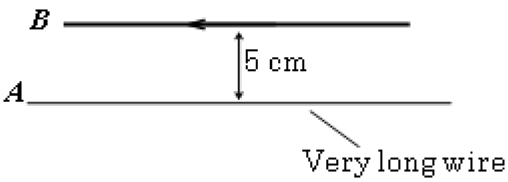
1. Two long parallel wires 10 cm apart are carrying currents of 3.0 A and 2.5 A in the same direction in vacuum. Find the force per unit length between the wires. (**Ans.** $1.5 \times 10^{-5}\ Nm^{-1}$).

Solution

Using $\frac{F'}{l} = F = \frac{\mu_0 I_1 I_2}{2\pi a}$

$$F = \frac{4\pi \times 10^{-7} \times 3 \times 2.5}{2\pi \times 10 \times 10^{-2}} = 1.5 \times 10^{-5}\ Nm^{-1}$$

2.



The diagram above shows two straight parallel wires A and B placed 5 cm apart and carrying currents of 1.5 A and 2.0 A respectively in opposite directions. If wire B is 15 cm long, find the magnitude of force acting on it.

Solution

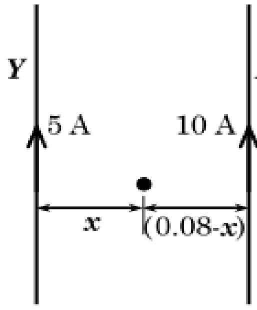
Using $F_b = \frac{\mu_0 I_a I_b l}{2\pi x}$

$$\Rightarrow F_b = \frac{4\pi \times 10^{-7} \times 1.5 \times 2.0 \times 15 \times 10^{-2}}{2\pi \times 5 \times 10^{-2}}$$

$$\therefore\ F_b = 1.8 \times 10^{-6}\ Nm^{-1}.$$

3. Two long parallel wires X and Y are separated by 8 cm in a vacuum. The wires carry current of 10 A and 5 A respectively in the same direction. At what point between the wires is the magnetic flux density zero?

Solution



Let the point be a distance x m from Y then (0.08-x) m from X.

From, $B = \frac{\mu_0 I}{2\pi a}$

$$B_y = \frac{4\pi \times 10^{-7} \times 5}{2\pi\, x}, B_x = \frac{4\pi \times 10^{-7} \times 10}{2\pi\,(0.08 - x)}.$$

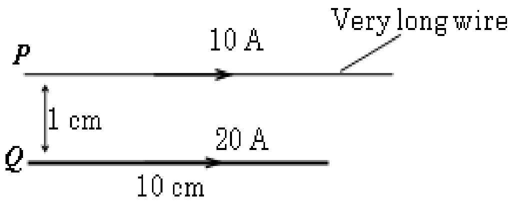
But at the neutral point, $B_y = B_x$

$$\Rightarrow \frac{4\pi \times 10^{-7} \times 5}{2\pi\, x} = \frac{4\pi \times 10^{-7} \times 10}{2\pi\,(0.08 - x)}$$

$$\Rightarrow \frac{5}{x} = \frac{10}{(0.08 - x)} \Leftrightarrow 10x = 0.4 - 5x$$

$$\Rightarrow 15x = 0.4 \quad \therefore \quad x = 0.02667m$$

4.



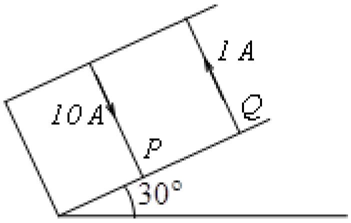
The figure above shows two parallel wires P and Q placed 1 cm apart and carrying currents of 10 A and 20 A respectively in the same direction. If wire Q is 10 cm long, find the force acting on Q. (**Ans.** 4.0×10^{-4} Nm $^{-1}$).

Using $F_q = \frac{\mu_0 I_p I_q l}{2\pi x}$

$$\Rightarrow F_q = \frac{4\pi \times 10^{-7} \times 10 \times 20 \times 10 \times 10^{-2}}{2\pi \times 1 \times 10^{-2}}$$

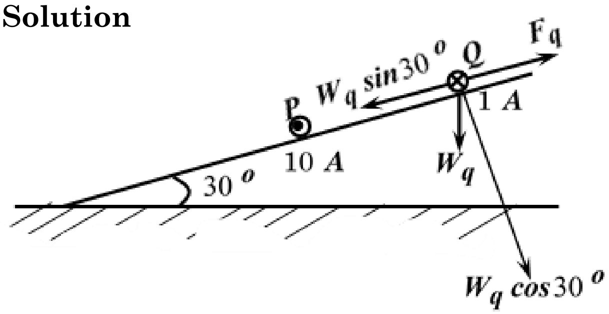
$$\therefore\ F_q = 4.0 \times 10^{-4}\ Nm^{-1}.$$

5. Two parallel wires ***P*** and ***Q*** each of length, 0.2 m carry currents of 10 A and 1 A respectively in opposite directions as shown below.



The distance between the wires is 0.04 m. If both wires remain stationary and the angle of the plane with the horizontal is 30^o, calculate the weight of ***Q***.

Solution



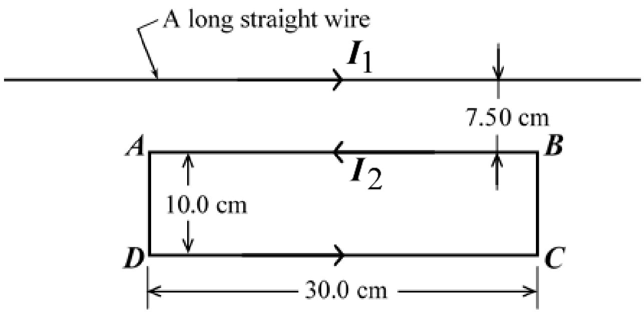
At equilibrium,

$$W_q \sin 30^o = F_q = \frac{\mu_0 I_p I_q l}{2\pi x}$$

$$\Rightarrow W_q = \frac{4\pi\times10^{-7}\times10\times1\times0.2}{2\pi\times0.04\times\sin 30^o}$$

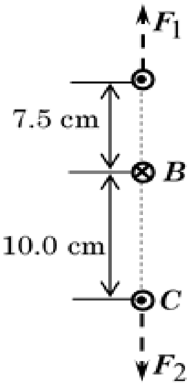
$\therefore$ Weight of ***Q***, $W_q = 2.0\times10^{-5}\,N$.

6. The figure below shows a long straight wire carrying current $I_1 = 15$ A placed near and parallel to a rectangular current loop ***ABCD*** of sides 30.0 cm **by** 10.0 cm carrying current $I_2 = 30$ A in a vacuum.



If the long straight wire is 7.50 cm away from side ***AB***, calculate the net force on side ***AB*** of the loop.

Solution



Wires carrying currents in opposite directions repel each other. Thus, the long wire repels side ***AB*** down with a force F_2 and side DC repels side ***AB*** upwards with a force F_1.

From, $F = \dfrac{\mu_0 I_1 I_2}{2\pi a}$

$$\Rightarrow F_1 = \frac{4\pi\times10^{-7}\times30\times30\times30\times10^{-2}}{2\pi\times10\times10^{-2}}$$

$\therefore\ F_1 = 5.4\times10^{-4}\,N$ and

$$F_2 = \frac{4\pi\times10^{-7}\times15\times30\times30\times10^{-2}}{2\pi\times7.5\times10^{-2}}$$

$\therefore\ F_1 = 3.6\times10^{-4}\,N$

The net force on side ***AB*** is

$$F = F_2 - F_1 = (5.4-3.6)\times10^{-4}$$

$F = 1.8\times10^{-4}\,N$ (Downwards).

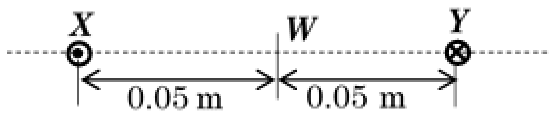
7. Two wires ***X*** and ***Y*** lie in a horizontal plane, their axes being 0.10 m apart. A current of 10 ***A*** flows in ***X*** in opposite direction to the current of 30 ***A*** in ***Y***. Neglecting the effect of the earth's magnetic field, determine the magnitude and direction of the flux at point ***W*** in the plane of the wires;

(a) mid-way between the wires,

(b) 0.05 m from wire ***X*** and 0.15 m from ***Y***

Solution

(a) When currents close to each other are in opposite direction, their magnetic fields produced reinforce each other in between the wires.



From, $B = \dfrac{\mu_0 I}{2\pi a}$

$$B_{yW} = \frac{4\pi \times 10^{-7} \times 30}{2\pi \times 0.05} = 1.2 \times 10^{-4} T \text{ and}$$

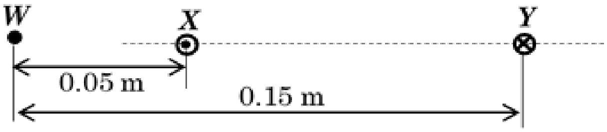
$$B_{xW} = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.05} = 0.4 \times 10^{-4} T.$$

The resultant magnetic flux density is

$$B_W = 0.4 \times 10^{-4} + 1.2 \times 10^{-4}$$

$$\therefore \; B_W = 1.6 \times 10^{-4} T \text{ (Upwards)}$$

(b)



At point W to the left of X the fields tend to cancel out each other since that due to current through Y is upwards and that due to current through X is downwards according to the right hand grip rule. Therefore,

From, $B = \dfrac{\mu_o I}{2\pi a}$

$$\Rightarrow B_{yW} = \frac{4\pi \times 10^{-7} \times 30}{2\pi \times 0.15} = 4.0 \times 10^{-5} T$$

(Upwards), and

$$B_{xW} = \frac{4\pi \times 10^{-7} \times 10}{2\pi \times 0.05} = 4.0 \times 10^{-5} T$$

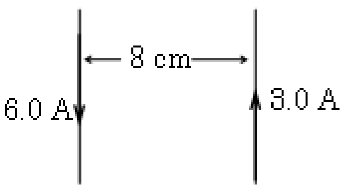
(Downwards).

The resultant magnetic flux density is

$$B_W = 4.0 \times 10^{-5} - 4.0 \times 10^{-5}$$

$$\therefore \; B_W = 0\, T.$$

8. Two long and parallel wires of negligible cross section area carry currents of 6.0 A and 3.0 A in opposite directions as shown below.



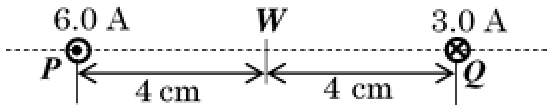
If the wires are separated by a distance of 8.0 cm, find the

(i) magnetic flux density at a point mid-way between the wires.

(ii) force per metre between the wires.(**Ans.** $4.5 \times 10^{-5} Nm^{-1}$).

Solution

(i) When currents close to each other are in opposite direction, their magnetic fields produced reinforce each other in between the wires



From, $B = \dfrac{\mu_o I}{2\pi a}$

$$B_{pW} = \frac{4\pi \times 10^{-7} \times 6}{2\pi \times 0.04} = 3.0 \times 10^{-5} T \text{ and}$$

$$B_{qW} = \frac{4\pi \times 10^{-7} \times 3}{2\pi \times 0.04} = 1.5 \times 10^{-5} T.$$

The resultant magnetic flux density is

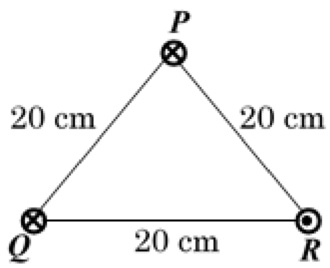
$$B_W = 3.0 \times 10^{-5} + 1.5 \times 10^{-5}$$

$$\therefore \; B_W = 4.5 \times 10^{-5} T \text{ (Upwards)}$$

(ii) Force per metre, $F = \dfrac{\mu_0 I_1 I_2}{2\pi a}$

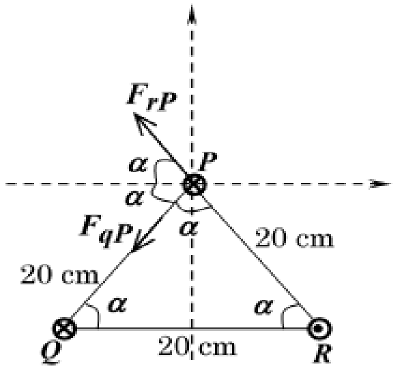
$$F = \frac{4\pi \times 10^{-7} \times 6 \times 3}{2\pi \times 8 \times 10^{-2}} = 4.5 \times 10^{-5} Nm^{-1}.$$

9. Three conductors P, Q and R carrying currents $3A$, $6A$ and $8A$ respectively are arranged as shown below



Find the resultant force per meter on conductor P.

Solution



By rules of angles in a triangle,

$3\alpha = 180^o \quad \Rightarrow \alpha = 60^o$

From, $F = \dfrac{\mu_0 I_1 I_2}{2\pi a}$

$$\Rightarrow F_{rP} = \frac{4\pi\times10^{-7}\times3\times6}{2\pi\times20\times10^{-2}} = 1.8\times10^{-5}N$$

and $F_{qP} = \dfrac{4\pi\times10^{-7}\times3\times8}{2\pi\times20\times10^{-2}} = 2.4\times10^{-5}N$

$$F_P = \begin{pmatrix} -1.8\times10^{-5}\cos60^o \\ 1.8\times10^{-5}\sin60^o \end{pmatrix} + \begin{pmatrix} -2.4\times10^{-5}\cos60^o \\ -2.4\times10^{-5}\sin60^o \end{pmatrix}$$

$$\Rightarrow F_P = \begin{pmatrix} -9.0\times10^{-6} \\ 1.55885\times10^{-5} \end{pmatrix} + \begin{pmatrix} -1.2\times10^{-5} \\ -2.07846\times10^{-5} \end{pmatrix}$$

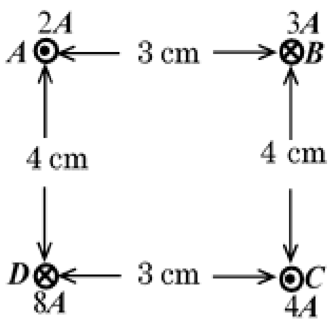
$$\Rightarrow F_P = \begin{pmatrix} -2.1\times10^{-5} \\ -5.1961\times10^{-6} \end{pmatrix}$$

The resultant force is

$$|F_P| = \sqrt{((2.1\times10^{-5})^2 + (5.1961\times10^{-6})^2)}$$

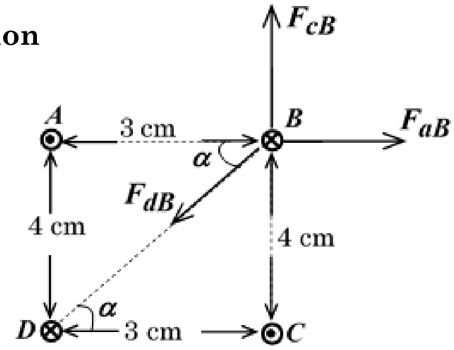
$\therefore\ |F_P| = 2.1633\times10^{-5}N$ acting at an angle of 13.9^o to the $2.1\times10^{-5}N$ component.

10. Four wires ***A, B, C*** and ***D*** carry currents of 2A, 3A, 4A and 8A respectively in free space as shown below.



Find the resultant force on ***B***

Solution



From triangle ***BDC,*** $\overline{DC} = \sqrt{(3^2+4^2)} = 5cm$

From triangle ***BDC***, $\alpha = \tan^{-1}\left(\dfrac{4}{3}\right) = 53.1^o$

From, $F = \dfrac{\mu_0 I_1 I_2}{2\pi a}$

$$\Rightarrow F_{aB} = \frac{4\pi\times10^{-7}\times3\times2}{2\pi\times3\times10^{-2}} = 4.0\times10^{-5}N$$

$$F_{cB} = \frac{4\pi\times10^{-7}\times3\times4}{2\pi\times4\times10^{-2}} = 6.0\times10^{-5}N$$

and $F_{dB} = \dfrac{4\pi\times10^{-7}\times3\times8}{2\pi\times5\times10^{-2}} = 9.6\times10^{-5}N$

$$F_B = \begin{pmatrix} 4.0\times10^{-5}\cos0^o \\ 4.0\times10^{-5}\sin0^o \end{pmatrix} + \begin{pmatrix} 6.0\times10^{-5}\cos90^o \\ 6.0\times10^{-5}\sin90^o \end{pmatrix} + \begin{pmatrix} -9.6\times10^{-5}\cos53.1^o \\ -9.6\times10^{-5}\sin53.1^o \end{pmatrix}$$

$$\Rightarrow F_B = \begin{pmatrix} 4.0\times10^{-5} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 6.0\times10^{-5} \end{pmatrix} + \begin{pmatrix} -5.76403\times10^{-5} \\ -7.67697\times10^{-5} \end{pmatrix}$$

$$\Rightarrow F_B = \begin{pmatrix} -1.76403\times10^{-5} \\ -1.67697\times10^{-5} \end{pmatrix}$$

The resultant force is

$$|F_B| = \sqrt{((1.76403\times10^{-5})^2 + (1.67697\times10^{-5})^2)}$$

$\therefore\ |F_B| = 2.43393\times10^{-5}N$ acting at an angle of 43.6^o to the $1.76403\times10^{-5}N$

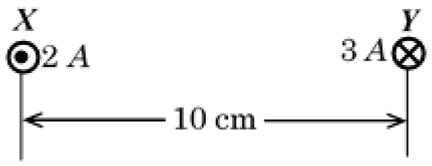
Trial questions

1. Two long parallel wires placed 12 cm apart in air carry currents of 10 A and 15 A respectively in the same direction. Determine the position where the magnetic flux is zero. (***Ans* 0.048 *m from a* 10*A current***)

2. Two long thin parallel wires A and B carry currents of 5A and 2A respectively in opposite direction. If the wires are separated by a distance of 2.5cm in a vacuum. Calculate the force exerted by wire B on 1m of wire A. (***Ans*** $8.0\times10^{-5}\,Nm^{-1}$)

3. Two parallel wires each of length 75 cm are placed 1.0 cm apart. When the same current is passed through the wires, a force of $5.0\times10^{-5}\,N$ develops between the wires. Find the magnitude of the current.
(***Ans*** $1.825742A$)

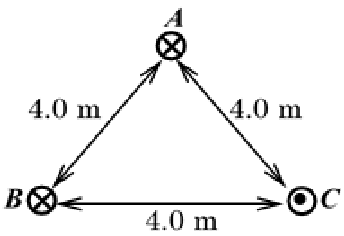
4. A long straight conductor X carrying a current of 2A is placed parallel to a short conductor, y of length 0.05m carrying a current of 3A as shown below



The two conductors are 0.10m apart.

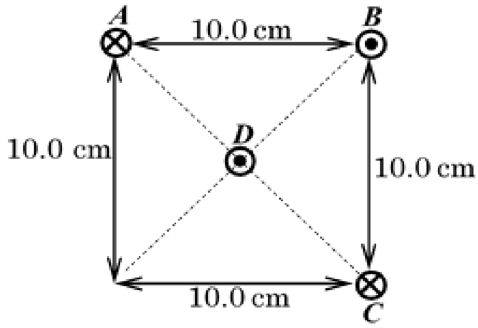
(i) Calculate the flux density due to ***X*** on ***Y*** (***Ans*** $4.0\times10^{-6}\,T$)
(ii) Determine approximate force on ***Y*** if it is 0.05 m long. (***Ans*** $6.0\times10^{-6}\,N$)
(iii) At what point between the wire is the magnetic flex density equal to zero? (***Ans* 0.04 *m from conductor X***)

5. Three long thin and parallel wires ***A***, ***B*** and ***C*** are fixed at the corners of an equilateral triangle in vacuum and carrying currents of 4.0 *A*, 2.0 *A* and 3.0 *A* respectively as shown below.



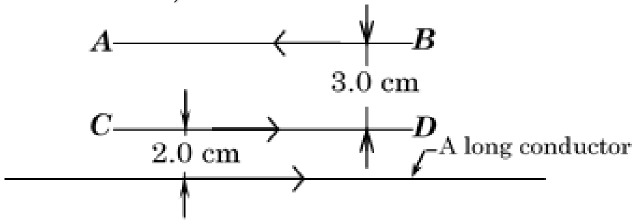
Calculate the resultant force per metre on wire ***A***. (***Ans*** $5.291502\times10^{-7}\,Nm^{-1}$ ***at an angle of*** 19.1^o ***to the horizontal***)

6. Three long thin and parallel wires ***A***, ***B*** and ***C*** are fixed at the corners of a square in a vacuum and carrying currents of 4.0 *A*, 2.0 *A* and 5.0 *A* respectively as shown below.



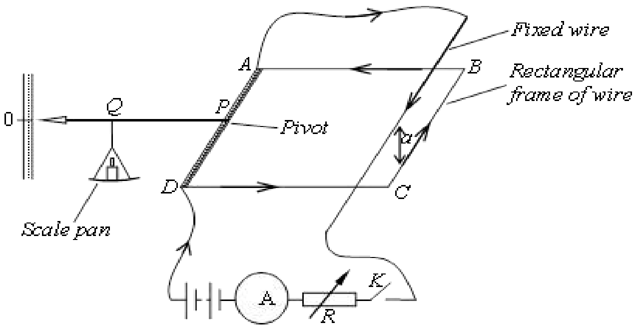
A short wire ***D*** of length 0.04 m and carrying a current of 3.0 *A* is fixed at the centre of the square of side 10 cm. Determine the resultant force on the entire wire ***D***.
(***Ans*** $7.589465\times10^{-7}\,N$ ***at an angle of*** 26.6^o ***to the force on D due to B***)

7. The figure below shows two wires ***AB*** and ***CD*** each carrying a current of 10.0 *A* in the direction shown. A long conductor carrying a current of 15*A* is placed parallel to the wire ***CD***, 2.0 cm below it.



(a) Calculate the net force on the long wire.
(b) Sketch the magnetic field pattern between the long wire and wire ***CD*** after removing wire ***AB***. Use the field pattern to define a magnetic neutral point.

Current balance (Absolute determination of current)

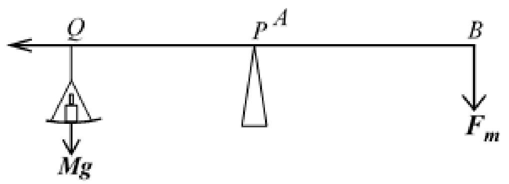


With no current flowing, the zero screw is adjusted until the conducting frame ***ABCD*** is horizontal.
The current, ***I*** to be measured is then passed through the circuit by closing switch, ***K*** such that the current through ***BC*** is in opposite direction with that flowing through $B^I C^I$.
Wire ***BC*** is thus repelled downwards.
Rider masses are therefore added to the scale pan until the horizontal balance is restored.
The mass, ***M*** on the scale pan is weighed and recorded.
The distance, a between the wires ***BC*** and $B^I C^I$, and the length, ***l*** of wire ***BC*** are measured and recorded.
The current, ***I*** is then calculated from,

$I = \sqrt{\left(\frac{Mga}{2\times10^{-7} l}\right)}$, where g is acceleration due to gravity.

Theory of the experiment.

If the rectangular frame of wire ***ABCD*** is horizontal when a current ***I***, is flowing through it then,



Taking moments about the pivot, ***P***,
$F_m \times \overline{AB} = Mg \times \overline{PQ}$. If $\overline{AB} = \overline{PQ}$ then,
$F_m = Mg$. But from, $F_m = BIl$ and also,

$$B = \frac{\mu_0 I}{2\pi a} \Rightarrow \frac{\mu_0 I^2 l}{2\pi a} = Mg$$

$$\Rightarrow I^2 = \frac{2\pi Mga}{\mu_0 l} = \frac{2\pi Mga}{4\pi\times10^{-7} l}$$

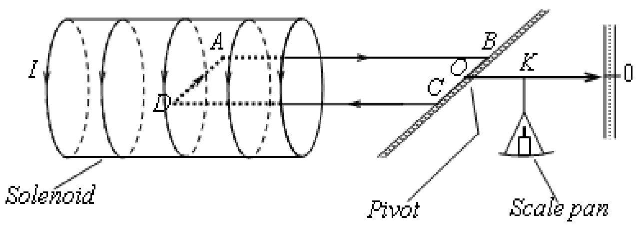
$$\therefore I = \sqrt{\left(\frac{Mga}{2\times10^{-7} l}\right)}$$

NB:

(i) Since the force between ***BC*** and $B^I C^I$ remain repulsive even when current is reversed thus, the **current** or **ampere balance** can be used to measure both D.C and A.C.

(ii) The current balance can be used to determine the magnitude of magnetic flux density at a point away from the wire carrying currents at the centre of a coil or along the axis of a long solenoid.

Using a solenoid to determine current, I.

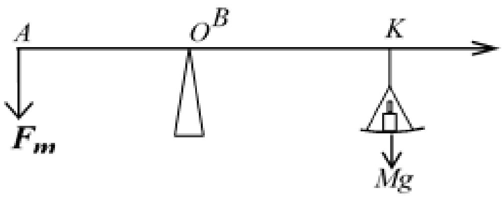


When there is no current, the zero screw is adjusted until the conducting frame ***ABCD*** is balanced horizontally.
The current, ***I*** to be determined is then passed through the circuit such that the current through the solenoid is opposite direction to the current through the conducting frame. As a result, ***AD*** is repelled downwards.
Rider masses are added to the scale pan until the horizontal balance is restored.
The mass, ***M*** of the scale pan is weighed and recorded.
The length, ***l*** of wire ***AD***, the distance ***AB*** and the distance ***OK*** are measured and recorded.
The current, ***I*** is then calculated from,

$I = \sqrt{\left(\frac{Mg\times\overline{OK}}{\mu_0 nl \times \overline{AB}}\right)}$, where g is acceleration due to gravity and n is the number of turns per metre of the solenoid.

Theory of the experiment.

If the rectangular frame of wire ***ABCD*** is horizontal when a current ***I***, is flowing through it then,



Taking moments about the pivot, ***O***,

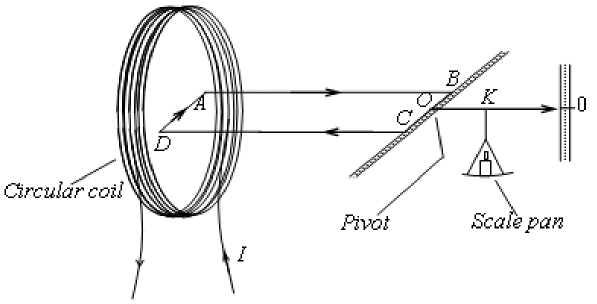
$F_m \times \overline{AB} = Mg \times \overline{OK}$. But from,

$F_m = BIl$ and $B = \mu_0 nI$.

$\Rightarrow \mu_0 nI^2 l \times \overline{AB} = Mg \times \overline{OK}$

$\therefore I = \sqrt{\left(\dfrac{Mg \times \overline{OK}}{\mu_0 nl \times \overline{AB}}\right)}$

Using a coil to determine current, *I*.

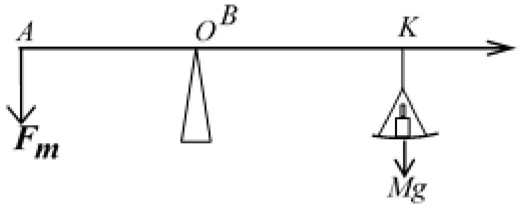


When there is no current, the zero screw is adjusted until the conducting frame ***ABCD*** is balanced horizontally.
The current, ***I*** to be determined is then passed through the circuit such that the current through the solenoid is opposite direction to the current through the conducting frame. As a result, ***AD*** is repelled downwards.
Rider masses are added to the scale pan until the horizontal balance is restored.
The mass, ***M*** of the scale pan is weighed and recorded.
The length, ***l*** of wire ***AD***, the distance ***AB*** and the distance ***OK*** are measured and recorded.
The mean radius, ***R*** of the coil is measured and recorded. The number, ***N*** of turns of the coil is also noted.
The current, ***I*** is then calculated from,

$I = \sqrt{\left(\dfrac{2RMg \times \overline{OK}}{\mu_0 N\, l \times \overline{AB}}\right)}$, where ***g*** is acceleration due to gravity.

Theory of the experiment.

If the rectangular frame of wire ***ABCD*** is horizontal when a current ***I***, is flowing through it then,



Taking moments about the pivot, ***O***,

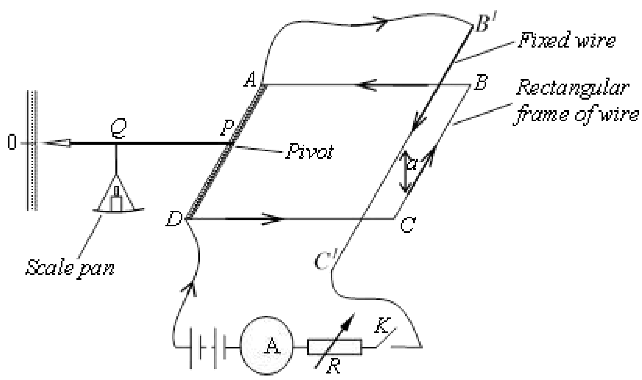
$F_m \times \overline{AB} = Mg \times \overline{OK}$. But from,

$F_m = BIl$ and $B = \dfrac{\mu_0 NI}{2R}$.

$\Rightarrow \dfrac{\mu_0 NI^2 l \times \overline{AB}}{2R} = Mg \times \overline{OK}$

$\therefore I = \sqrt{\left(\dfrac{2RMg \times \overline{OK}}{\mu_0 N\, l \times \overline{AB}}\right)}$.

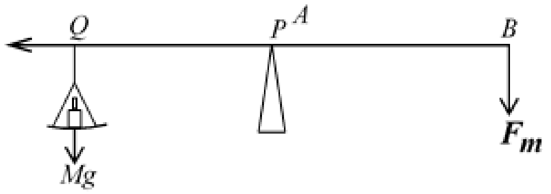
An experiment to determine magnetic flux density at a point away from a wire carrying current.



With no current flowing, the zero screw is adjusted until the conducting frame ***ABCD*** is horizontal.
The current, ***I*** to be measured is then passed through the circuit by closing switch, ***K*** such that the current through ***BC*** is in opposite direction with that flowing through $B^I C^I$.
Wire ***BC*** is thus repelled downwards.
Rider masses are therefore added to the scale pan until the horizontal balance is restored.
The mass, ***M*** on the scale pan is weighed and recorded.
The current, I through the circuit is read and recorded.

The length, ***l*** of wire ***BC*** are measured and recorded.
The magnetic flux density, ***B*** is then calculated from, $B = \frac{Mg}{I\,l}$, where ***g*** is acceleration due to gravity.

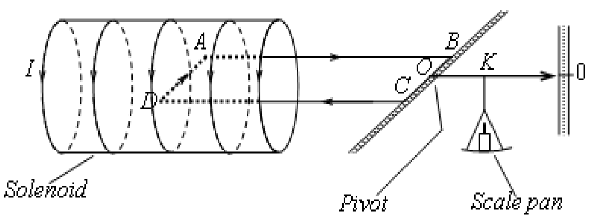
Theory of the experiment.



If the rectangular frame of wire ***ABCD*** is horizontal when a current ***I***, is flowing through it then,

Taking moments about the pivot, ***P***,
$F_m \times \overline{AB} = Mg \times \overline{PQ}$. If $\overline{AB} = \overline{PQ}$ then,
$F_m = Mg$. But from, $F_m = BIl$.
$\Rightarrow BIl = Mg \quad \therefore\ B = \frac{Mg}{I\,l}$.

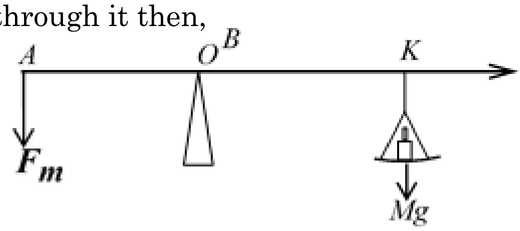
Using a solenoid to determine magnetic flux density, B.



When there is no current, the zero screw is adjusted until the conducting frame ***ABCD*** is balanced horizontally.
A known current, ***I***, is then passed through the circuit such that the current through the solenoid is opposite direction to the current through the conducting frame. As a result, ***AD*** is repelled downwards.
Rider masses are added to the scale pan until the horizontal balance is restored.
The mass, ***M*** of the scale pan is weighed and recorded.
The length, ***l*** of wire ***AD***, the distance ***AB*** and the distance ***OK*** are measured and recorded.
The magnetic flux density, ***B*** is then calculated from, $B = \frac{Mg \times \overline{OK}}{I\,l \times \overline{AB}}$.

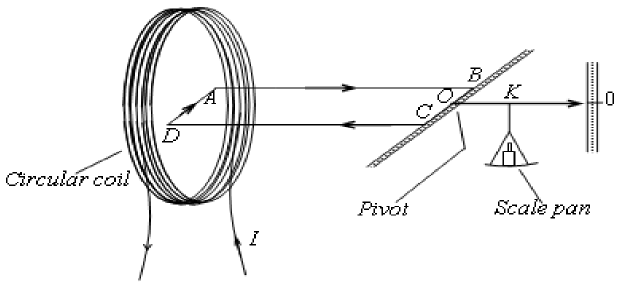
Theory of the experiment.

If the rectangular frame of wire ***ABCD*** is horizontal when a current ***I***, is flowing through it then,



Taking moments about the pivot, ***O***,
$F_m \times \overline{AB} = Mg \times \overline{OK}$. But from,
$F_m = BIl$.
$\Rightarrow BIl = Mg \quad \therefore\ B = \frac{Mg \times \overline{OK}}{I\,l \times \overline{AB}}$.

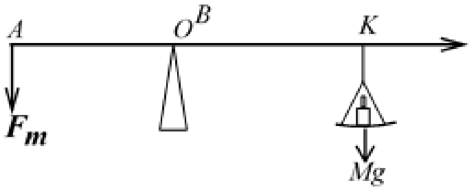
Using a coil to determine current, I.



When there is no current, the zero screw is adjusted until the conducting frame ***ABCD*** is balanced horizontally.
The current, ***I*** to be determined is then passed through the circuit such that the current through the solenoid is opposite direction to the current through the conducting frame. As a result, ***AD*** is repelled downwards. Rider masses are added to the scale pan until the horizontal balance is restored.
The mass, ***M*** of the scale pan is weighed and recorded.
The length, ***l*** of wire ***AD***, the distance AB and the distance ***OK*** are measured and recorded.
The magnetic flux density, ***B*** is then calculated from, $B = \frac{Mg \times \overline{OK}}{I\,l \times \overline{AB}}$, where ***g*** is acceleration due to gravity.

Theory of the experiment.

If the rectangular frame of wire ***ABCD*** is horizontal when a current ***I***, is flowing through it then,

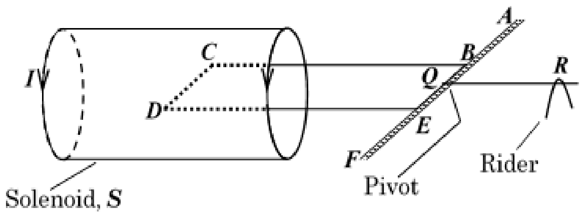


Taking moments about the pivot, ***O***,
$F_m \times \overline{AB} = Mg \times \overline{OK}$. But from,
$F_m = BIl$.

$$\Rightarrow BIl = Mg \quad \therefore\ B = \frac{Mg \times \overline{OK}}{I\,l \times \overline{AB}}.$$

NUMERICAL EXAMPLES

1. The figure shows a simple form of a current balance.



A long solenoid, ***S*** which has 2000 turns per metre is in series with a horizontal rectangular copper loop ***ABCDEF*** where ***BC*** = 12 cm and ***CD*** = 5 cm. The loop which is freely pivoted on the axis ***BE*** goes well inside the solenoid and ***CD*** is perpendicular to the axis of ***S***. When the current is switched on, a rider of mass 0.25 g placed 6 cm from the axis ***BE*** is needed to restore equilibrium. Calculate the value of ***I***.

Solution

By principle of moments,

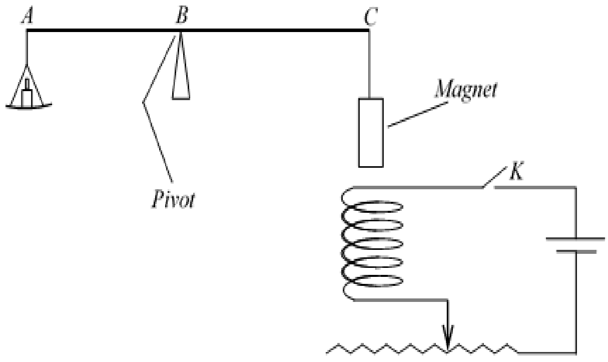
$F_m \times \overline{CB} = mg \times \overline{QR}$, where,
$F_m = BI_1L = BI_1 \times \overline{CD}$ and
$B = \mu_0 nI = 4\pi \times 10^{-7} \times 2000 \times I$
$B = \mu_0 nI = 2.513274 \times 10^{-3} I$. But $I_1 = I$
Thus,

$$2.513274 \times 10^{-3} I^2 \times 0.05 \times 0.12 = 0.00025 \times 9.81 \times 0.06$$

$$I = \sqrt{\left(\frac{0.00025 \times 9.81 \times 0.06}{2.513274 \times 10^{-3} \times 0.05 \times 0.12}\right)}$$

$I = 3.123810\,A$

2.



The figure above represents a current balance. When switch K is open, the force required to balance is 0.2 N. When switch K is closed and the current of 0.5 A flows, a force of 0.22 N is required for balance. Determine the

(i) polarity at the end of the magnet closest to the coil.
(ii) weight required for balance when a current of 2 A flows through the coil.

Solution

(i) When the switch is closed, current flows through the solenoid and by the ***right hand grip rule***, the thumb points upwards and so, the end of the coil close to the magnet is a ***north pole.*** Masses are added to the pan to restore horizontal balance if the magnet is pulled downwards (attracted). Therefore, the end of the magnet near the coil is a south pole for attraction between the coil and the magnet to occur.

(ii) For 0.5*A*, the magnetic force due to the coil is $F_m = F_r - W_m$, where F_r is the force required for balance and W_m is weight of the magnet.

$F_m = 0.22 - 0.2 = 0.02N$

But, $F_m \propto I^2 \Rightarrow F_m = kI^2$

$\Rightarrow k = \frac{F_m}{I^2} = \frac{0.02}{0.5}\ \therefore\ k = 0.08NA^{-2}$

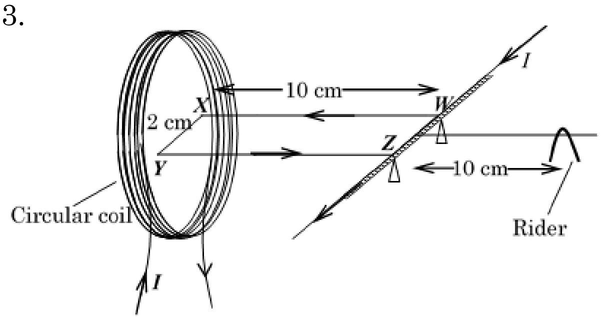
For 2*A*, the magnetic force is

$F_m = 0.08 \times 2^2 = 0.32N$

Therefore, force required is

$F_r = 0.22 + 0.32 = 0.54N$.

3.



A rectangular loop of wire ***WXYZ*** is balanced horizontally so that the length ***XY*** is at the centre of a circular coil of 500 turns of mean radius 10.0 cm as shown. When current, ***I*** is passed through ***XY*** and the circular coil, a rider of mass $5.0\times10^{-4}\,kg$ has to be placed at a distance of 9.0 cm from ***WZ*** to restore balance. Find the value of current, ***I***.

Solution

By principle of moments,

$F_m \times \overline{XW} = mg\times 0.09$, where,

$F_m = BIL = BI\times\overline{XY}$ and

$$\Rightarrow B = \frac{\mu_o NI}{2R} = \frac{4\pi\times10^{-7}\times500\times I}{2\times10\times10^{-2}}$$

$\therefore B = 3.141593\times10^{-3}I$. Thus,

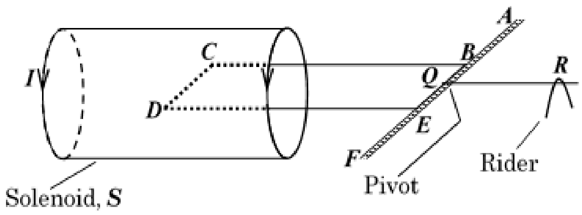
$$3.141593\times10^{-3}I^2\times0.02\times0.1 = 0.0005\times9.81\times0.09$$

$$I = \sqrt{\left(\frac{0.0005\times9.81\times0.09}{3.141593\times10^{-3}\times0.02\times0.1}\right)}$$

$I = 8.382061\,A$

<u>Trial questions</u>

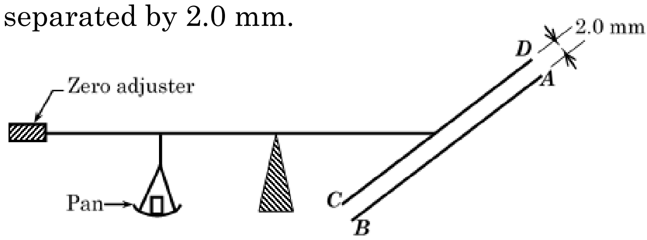
1. The figure shows a simple form of a current balance.



A long solenoid, ***S*** which has 2000 turns per metre is in series with a horizontal rectangular copper loop ***ABCDEF*** where ***BC*** = 10.0 cm and ***CD*** = 3.0 cm. The loop which is freely pivoted on the axis ***AF*** goes

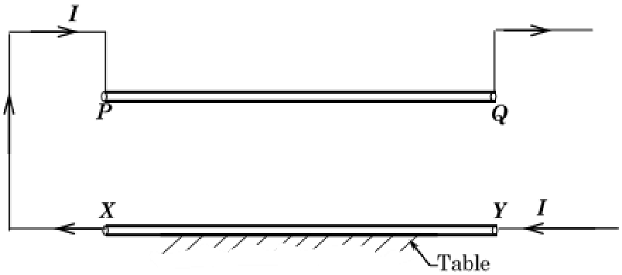
well inside the solenoid and ***CD*** is perpendicular to the axis of ***S***. When the current is switched on, a rider of mass 0.2 g placed 5.0 cm from the axis is needed to restore equilibrium. Calculate the value of ***I***. (***Ans*** $3.607065\,A$)

2. The figure below shows an ampere balance. Wires ***AB*** and ***CD*** each of length 100 cm, lie in the same vertical plane and are separated by 2.0 mm.



When current, ***I*** is passed in opposite directions through the wires, a mass of 0.3 g is placed in the pan to obtain balance. Find the value of the current ***I*** if the pan and the wire ***AB*** are 5 cm and 8 cm from the pivot respectively. (***Ans*** $4.288794\,A$)

3. A wire ***XY*** rests on a horizontal non conducting table and another wire ***PQ*** of length 12.0 cm is free to move vertically in guides at the end ***P*** and ***Q*** above ***XY*** as shown below.



The mass per unit length of ***PQ*** $3\,mg\,cm^{-1}$. A current of 3.6*A* through the wires was enough to maintain the ***PQ*** at the distance ***d*** *cm* from ***XY***.

(i) Calculate the distance of separation, ***d***

(ii) Find the magnetic flux density due to ***PQ*** on ***XY***.

4. Two thin horizontal rods, ***XY*** and ***PQ*** each 1.0 ***m*** in length carrying currents of equal magnitudes and are connected so that ***PQ*** is located 0.5 ***cm*** above ***XY***. The lower rod is fixed while the upper rod is kept in equilibrium by magnetic repulsion. The mass of each rod is $1.0\times10^{-2}\,kg$. Determine the value of current in each rod.